Homework 05

1. Section 10.8: problem 40.
2. Section 10.8: Problem 40 says that $g(x)$ is the only possible candidate that satisfies (i), (ii). Show that, if $f^{(n+1)}(x)$ is continuous on $[a-\delta, a+\delta]$, $\delta > 0$, then $g(x)$ indeed satisfies (i), (ii).
3. Section 10.8: Use Theorem A to solve Problems 5 ($n = 3$), 35, 41.

   Remark:

   Method 1: (Should be more complicated in general): Find $f^{(n)}(a)$ by repeated differentiation. This should be the method you used last week.

   Method 2: Apply Theorem A (should be simpler in general).

   Hint:

   For problem 5: Find geometric series representation of $\dfrac{1}{x} = \dfrac{1}{x-2+2} = \dfrac{1}{2}\dfrac{1}{\left(1+\dfrac{x-2}{2}\right)}$ and apply Theorem A.

   For problem 35: Apply Theorem 19 and Theorem A.

   For problem 41: Apply Theorem 20 and Theorem A.
4. Section 10.9: Problems 7, 10, 17, 19, 31, 33, 41, 42, 50(a), 51. Most of them are implicitly using Theorem A (i.e. problem 51).
5. Section 10.10: Problems 10, 19, 31, 35, 37, 45, 51, 58, 65, 66, 72, 73.

Theorem A (See also section 10.9, problem 51):
If $f(x)$ has a power series representation on $|x-a| < R$ with $\underline{R > 0}$, then $T_{f,a}(x) = f(x)$.

In other words:

If $f(x) = \sum_{k=0}^{\infty} a_k (x-a)^k$ on $|x-a| < R$ with $\underline{R > 0}$, then $T_{f,a}(x) = \sum_{k=0}^{\infty} a_k (x-a)^k$.