Approximation Algorithms for Data Center Networks Design

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The network design of the data center has a huge impact on the computing power of large-scale artificial intelligence language models. In this talk we introduce two data center network design issues. Given a metric graph $G = (V, E, w)$, a center $c \in V$, and an integer k, **the Star k-Hub Center Problem** is to find a depth-2 spanning tree T of G rooted by c such that c has exactly k children and the diameter of T is minimized. Those children of c in T are called hubs. A similar problem called the **Single Allocation k-Hub Center Problem** is to find a spanning subgraph H^* of G such that (i) C^* is a clique of size k in H^*; (ii) $V \backslash C^*$ forms an independent set in H^*; (iii) each $v \in V \backslash C^*$is adjacent to exactly one vertex in C^*; and (iv) the diameter $D(H^*)$ is minimized. The vertices selected in C^* are called hubs and the rest of vertices are called non-hubs. Both **Star k-Hub Center Problem** and **Single Allocation k-Hub Center Problem** are NP-hard and have applications in transportation system, telecommunication system, and post mail system. In this talk, we give 5/3-approximation algorithms for both problems. Moreover, we prove that for any $\varepsilon > 0$, the **Star k-Hub Center Problem** has no $(1.5 - \varepsilon)$-approximation algorithm unless P = NP. Under the assumption P $\neq$ NP, for any $\varepsilon > 0$ the **Single Allocation k-Hub Center Problem** has no $(4/3 - \varepsilon)$-approximation algorithm.