CHAPTER 1, FUNCTIONS, GRAPHS AND MODELS

1.1 Functions and Mathematical Modelings

Functions.

Functions. *A real valued function f defined on a set of real numbers D is a rule that assigns each $x \in D$ exactly one real number denoted by $f(x)$. D is the domain of f, and $f(D) = \{y : y = f(x), x \in D\}$ the range of f .*

Example.

(1) $f(x) = x^2$,
(2) $f(x) = x^2 + x - 3$,
(3) $h(x) = x^2, x \geq 0, = \sqrt{-x}, x < 0$
(4) $[[x]] = n, n \leq x < n + 1$,
(5) FLOOR$(x) = [[x]]$, ROUND$(x) = [[x + 1/2]]$, CEILING$(x) = -[[-x]]$,
(6) $\sin x$, $\sin^{-1} x$.

Domain, Intervals.

Example.

(1) $g(x) = \frac{1}{2x+4}$,
(2) $h(x) = \frac{1}{\sqrt{2x+4}}$.

Mathematical Modelings.

Example.

(1) Example 8 0n page 5 $V = x^2y = 125, A = 4xy + 2x^2$,
(2) Example 9 0n page 6 $A = xy, 180 = 6x + 10y$.

1.2 Graphs of Equations and Functions

Graph of Equation. *The graph of an equation of two variables x and y is the set of points satisfy the equation.*

$$f(x, y) = c.$$

Example. $(x - 3)^2 + (y - 4)^2 = 100$

Translation principle.
Graph$(f(x, y) = 0) + (h, k) =$*Graph*$(f(x - h, y - k) = 0)$

Example. $x^2 + y^2 - 4x + 6y = 12$

Graphs of Functions. *The graph of a function is the graph of* $y = f(x)$*.*

Example.

(1) $|x|$,
(2) $f(x) = \frac{1}{x}$,
(3) $[[x]]$,
(4) $g(x) = x - [[x]] - \frac{1}{2}$,
(5) $f(x) = x^2$,
(6) $g(x) = \sqrt{x}$.

Remark. $y = ax^2 + bx + c$ *can be written as* $y - c + \frac{b^2}{4a} = a(x + \frac{b}{2a})^2$.

(1) $y = 2x^2 - 4x - 1$ is $y + 3 = 2(x-1)^2$,
(2) $A = \frac{3}{5}(30x - x^2)$ is $A - 135 = -\frac{3}{5}(x - 15)^2$.

Remark. *A graph* G *is a graph of a function on* $[a, b]$ *if and only if every vertical line* $x = c : c \in [a, b]$ *meets* G *in exactly one point.*

1.3 Algebraic Functions

Power Functions. $x^n, x^{\frac{p}{q}}$.

Combination of functions.

(1) $cf(x) = c(f(x))$,
(2) $(f + g)(x) = f(x) + g(x)$.

Example.

(1) $f(x) = x^2 + 1, g(x) = x - 1$,
(2) $f(x) = \sqrt{1-x}, g(x) = \sqrt{1+x}$.

Polynomials. $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$.

Rational Functions. $f(x) = \frac{p(x)}{q(x)}$.

Example.

(1) $\frac{(x+2)(x-1)}{x(x+1)(x-2)}$,
(2) $\frac{x(x+1)(x-2)}{(x+2)(x-1)}$.

Algebraic Functions.
$(16 - x^4)^{1/4}$, $\sqrt{x^2 - 16}$, $\sqrt{x^2}$, $(x^2(x-2)^2)^{1/3}$.

1.4 Transcendental Functions

Transcendental functions. *Any function which is not algebraic function.*

Elementary transcendental functions.

Trigonometric Functions. $\sin x$, $\cos x$, $\tan x$ *etc..*

Composition of Functions. *The composition of two functions f and g , $h = f \circ g$ is defined by $h(x) = f(g(x)))$ for all x in the domain of g such that $g(x)$ is in the domain of f.*

Example.

(1) $f(x) = \sqrt{x}, g(x) = 1 - x^2$,
(2) $f(x) = x^2, g(x) = \cos x.$, $h(x) = (x^2 + 2)^{3/2}$.

Exponential Functions. $a > 0,\ a^x$.

$32^{-t} \cos 4\pi t$

Inverse Function. *$g(y)$ is the inverse function of $f(x)$ provided that $g(y) = x$ if and only if $f(x) = y$.*

Example. $\sin^{-1} x, \cos^{-1} x$.

Logarithmic Function. $y = \log_a x$ *if and only if* $a^y = x$.

Implicit function. *From $F(x, y) = 0$ define y is a function of x.*

Transcendental equation. $x = \tan x$

Can you brlieve what you see ?. $y = \sin 120\pi x$.