Calculus (II) — Homework 6 (Fall 2026)

1. Expand $f(x)$ in powers of x.

   (a) $f(x) = \dfrac{1}{(1-x)^2}$.

   (b) $f(x) = \ln(2-3x)$.

   (c) $f(x) = \sin(x^2)$.

   (d) $f(x) = x^2 \arctan x$.

   (e) $f(x) = \dfrac{1}{1-x} + e^{2x^3}$.

   (f) $f(x) = \cosh x \sinh x$.

2. Find $f^{(9)}(0)$.

   (a) $f(x) = \ln(2-3x)$.

   (b) $f(x) = x\sin(x^2)$.

   (c) $f(x) = x^2 \arctan x$.

   (d) $f(x) = \dfrac{1}{1-x} + e^{2x^3}$.

3. Evaluate the limit by using power series.

   (a) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$.

   (b) $\lim\limits_{x\to 0} \dfrac{\sin x - x}{x^2}$.

4. Sum the series.

   (a) $\sum\limits_{k=0}^{\infty} \dfrac{1}{k!} x^{3k+1}$.

   (b) $\sum\limits_{k=0}^{\infty} \dfrac{3k}{k!} x^{3k-1}$.

5. Set $f(x) = \dfrac{e^x - 1}{x}$.

   (a) Expand $f(x)$ in a power series.

   (b) Differentiate the series and show that

   $$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1.$$

6. Suppose that the power series $\sum\limits_{k=0}^{\infty} a_k x^k$ and $\sum\limits_{k=0}^{\infty} b_k x^k$ converge to the same value for all $x \in (-r, r)$ with $r > 0$. Show that $a_k = b_k$ for all $k \geq 0$.

7. We say that a function $f(x)$ is **entire** if $f(x) = \sum\limits_{k=0}^{\infty} \dfrac{f^{(k)}(0)}{k!} x^k$ for all $x \in \mathbb{R}$. Find all the entire functions that satisfy the equation

   $$f''(x) + xf(x) = 0.$$