Calculus (II) — Homework 5 (Fall 2026)

1. Prove that, for $x \in (0, 1)$,
$$\ln(1 + x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k.$$

2. Expand $g(x)$ in the powers of $x - 1$ and specify the values of x for which the expansion is valid.

   (a) $g(x) = 3x^3 - 2x^2 + 4x + 1$.
   (b) $g(x) = x^{-1}$.
   (c) $g(x) = e^{-4x}$.
   (d) $g(x) = \sin \pi x$.
   (e) $g(x) = \cos(\frac{1}{2}\pi x)$.
   (f) $g(x) = \ln(1 + 2x)$.

3. Suppose that the series $\sum_{k=0}^{\infty} a_k(-3)^k$ converges. What can you conclude about the convergence of the following series?

   (a) $\sum_{k=1}^{\infty} a_k 2^k$.
   (b) $\sum_{k=2}^{\infty} a_k 3^k$.
   (c) $\sum_{k=3}^{\infty} (-1)^k |a_k|$.
   (d) $\sum_{k=4}^{\infty} a_k 4^k$.

4. Find the interval of convergence.

   (a) $\sum_{k=1}^{\infty} k x^k$.
   (b) $\sum_{k=1}^{\infty} \frac{1}{(2k)!} x^{4k}$.
   (c) $\sum_{k=1}^{\infty} (-k)^{2k} x^{2k}$.
   (d) $\sum_{k=1}^{\infty} \frac{1}{k 2^k} x^k$.
   (e) $\sum_{k=1}^{\infty} \frac{\ln k}{k} (x + 1)^k$.
   (f) $\sum_{k=1}^{\infty} (-1)^k \frac{k!}{k^3} (x - 1)^k$.
   (g) $\sum_{k=1}^{\infty} \frac{\ln k}{2^k} (x - 2)^k$.
   (h) $\sum_{k=2}^{\infty} \frac{1}{(\ln k)^k} (x - 1)^k$.
   (i) $\sum_{k=1}^{\infty} \left(1 + \frac{1}{k}\right)^k (x + 2)^k$.