Calculus (II) — Homework 3 (Fall 2026)

1. Differentiate the following functions.

   (a) $f(x) = \arctan(x+1)$ (b) $f(x) = e^x \arcsin x$

2. Let $a > 0$. Calculate the indefinite integral:

$$\int \frac{1}{a^2 + (x+b)^2}\, dx.$$

3. Let $f(u)$ be a continuous function, let $u(x)$ be a continuously differentiable function, and let $F(u)$ be an antiderivative of $f(u)$ (i.e., $F'(u) = f(u)$). Recall the Chain Rule:

$$\frac{d}{dx}F(u(x)) = F'(u(x)) \cdot u'(x) = f(u(x)) \cdot u'(x).$$

   (a) Use the Fundamental Theorem of Calculus and the Chain Rule above to prove the Substitution Rule for definite integrals:

$$\int_a^b f(u(x)) \cdot u'(x)\, dx = \int_{u(a)}^{u(b)} f(u)\, du.$$

   (b) Evaluate the following definite integrals:

   (i) $\displaystyle\int_0^1 (x^2+1)^5 x\, dx$ (ii) $\displaystyle\int_1^2 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

4. Let $u(x)$ and $v(x)$ be continuously differentiable functions. Recall the Product Rule:

$$\frac{d}{dx}[u(x)v(x)] = u'(x)v(x) + u(x)v'(x).$$

   (a) Use the Fundamental Theorem of Calculus and the Product Rule above to prove the Integration by Parts formula for definite integrals:

$$\int_a^b u(x)v'(x)\, dx = u(x)v(x)\big|_a^b - \int_a^b v(x)u'(x)\, dx.$$

   (b) Evaluate the following definite integrals:

   (i) $\displaystyle\int_0^1 xe^{-x}\, dx$ (ii) $\displaystyle\int_1^{e^2} x \ln(\sqrt{x})\, dx$

5. Evaluate the following improper integrals.

   (a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}}$ (d) $\displaystyle\int_0^\infty xe^{-2x}\, dx$

   (b) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ (e) $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\, dx$

   (c) $\displaystyle\int_0^1 x \ln x\, dx$ (f) $\displaystyle\int_0^1 \frac{1}{x^2-1}\, dx$

6. Determine whether each improper integral converges or diverges.

(a) $\displaystyle\int_{1/2}^{2} \frac{dx}{x \ln x}$

(b) $\displaystyle\int_{0}^{\pi/2} \tan x \, dx$

(c) $\displaystyle\int_{1}^{\infty} \frac{dx}{x^3 + 1}$

(d) $\displaystyle\int_{1}^{\infty} \frac{\sqrt{x+1}}{x^2} \, dx$

(e) $\displaystyle\int_{\pi}^{\infty} \frac{2 + \cos x}{x} \, dx$

(f) $\displaystyle\int_{1}^{\infty} \frac{e^x}{x} \, dx$