Calculus (II) — Homework 1 (Fall 2026)

1. Determine the boundedness and monotonicity (i.e. increasing or decreasing or neither) of the sequence with a_n, as indicated.

   (a) $a_n = \dfrac{2}{n}$.

   (b) $a_n = \dfrac{(-1)^n}{n}$.

   (c) $a_n = \dfrac{n + (-1)^n}{n}$.

   (d) $a_n = \sqrt{n^2 + 1}$.

   (e) $a_n = \dfrac{2^n}{4^n + 1}$.

   (f) $a_n = \dfrac{1}{2n} - \dfrac{1}{2n+3}$.

   (g) $a_n = \ln\left(\dfrac{n+1}{n}\right)$.

   (h) $a_n = \sin \dfrac{\pi}{n+1}$.

   (i) $a_n = \dfrac{3^n}{(n+1)^2}$.

2. Let a_n be the sequence satisfying the given rules. Find an explicit formula for a_n that does not involve an recursive relation. Explain your answer.

   (a) $a_1 = 1; \qquad a_{n+1} = \frac{1}{2}a_n + 1$.

   (b) $a_1 = 1; \qquad a_{n+1} = \dfrac{n}{n+1}a_n$.

   (c) $a_1 = 1; \qquad a_{n+1} = a_n + \dfrac{1}{n(n+1)}$.

   (d) $a_1 = 1; \qquad a_{n+1} = a_n + \cdots + a_1$.

3. Let r be a real number, and

$$a_n = 1 + r + r^2 + \cdots + r^{n-1}.$$

   (a) If $r = 1$, what is a_n for $n = 1, 2, 3, \cdots$?

   (b) If $r \neq 1$, what is a_n for $n = 1, 2, 3, \cdots$? Find a formula for a_n that does not involve adding up the powers of r.

   (c) For what values of r does a_n converge?

   (d) Find the limit $\lim\limits_{n\to\infty} a_n$ for $|r| < 1$.

4. State whether the sequence converges and, if it does, find the limit.

   (a) $a_n = 2^n$.

   (b) $a_n = \dfrac{(-1)^n}{n}$.

   (c) $a_n = \dfrac{n + (-1)^n}{n}$.

   (d) $a_n = \dfrac{4^n}{\sqrt{n^2+1}}$.

   (e) $a_n = \dfrac{2^n}{4^n + 1}$.

   (f) $a_n = \dfrac{1}{2n} - \dfrac{1}{2n+3}$.

   (g) $a_n = \ln\left(\dfrac{n+1}{n}\right)$.

   (h) $a_n = \sin \dfrac{\pi}{n+1}$.

   (i) $a_n = \dfrac{(n+1)(n+2)}{(n+3)(n+4)}$.