Calculus — Homework 12 (Spring 2026)

1. Find critical points and classify them.
   (a) $f(x, y) = x^2 + xy + y^2 - 6x + 2$.
   (b) $f(x, y) = x \sin y$.
   (c) $(x - 3)\ln(xy)$.

2. Find the absolute extreme values taken by f on the set indicated.
   (a) $f(x, y) = 2x^2 + y^2 - 4x - 2y + 2$; $\quad 0 \le x \le 2, 0 \le y \le 2x$.
   (b) $f(x, y) = (x - 3)^2 + y^2$; $\quad 0 \le x \le 4, x^2 \le y \le 4x$.
   (c) $f(x, y) = (x - 1)^2 + (y - 1)^2$; $\quad x^2 + y^2 \le 4$.

3. Find maxima or minima with side conditions.
   (a) Minimize $x^2 + y^2$; on the hyperbola $xy = 1$.
   (b) Maximize $x + y$ on the curve $x^4 + y^4 = 1$.
   (c) Minimize $x + 2y + 4z$ on the sphere $x^2 + y^2 + z^2 = 7$.
   (If you are not familiar with functions of three variables, you can find examples on page 846 in the textbook.)

4. Let x, y, z be the three angles of a triangle. Determine the maximum value of

$$f(x, y, z) = \sin x \sin y \sin z.$$

5. Let $f = f(x, y)$ be a smooth function on $\mathbb{R}^2$, and let H denote the matrix:

$$H = \begin{pmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{xy}(0,0) & f_{yy}(0,0) \end{pmatrix}.$$

   Suppose that $\lambda \in \mathbb{R}$ is an **eigenvalue** of H, i.e., there exists a nonzero vector $\vec{v} = (v_1, v_2) \in \mathbb{R}^2$ such that

$$H\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

   Prove the following statements:
   (a) If f has a local minimum at $(0, 0)$, then $\lambda \ge 0$.
   (b) If f has a local maximum at $(0, 0)$, then $\lambda \le 0$.

6. Let R be the rectangle $a \le x \le b$, $c \le y \le d$. Show that, if f is continuous on $[a, b]$ and g is continuous on $[c, d]$, then

$$\iint_R f(x)g(y)\, dxdy = \left(\int_a^b f(x)\, dx\right) \cdot \left(\int_c^d g(y)\, dy\right).$$

7. Evaluate the integral for $\Omega : 0 \le x \le 1, 0 \le y \le 3$.

   (a) $\displaystyle\iint_\Omega x^2\, dxdy$. $\qquad$ (b) $\displaystyle\iint_\Omega e^{x+y}\, dxdy$.

8. Evaluate the integral for $\Omega : 0 \le x \le 1, 0 \le y \le x$.

   (a) $\displaystyle\iint_\Omega x^3 y\, dxdy$. $\qquad$ (b) $\displaystyle\iint_\Omega x^2 y^2\, dxdy$.

9. Evaluate the double integral.

(a) $\iint\limits_{\Omega} (x + 3y^3)\, dxdy, \quad \Omega : 0 \leq x^2 + y^2 \leq 1.$

(b) $\iint\limits_{\Omega} \sqrt{xy}\, dxdy, \quad \Omega : 0 \leq y \leq 1, y^2 \leq x \leq y.$

(c) $\iint\limits_{\Omega} (4 - y^2)\, dxdy,$ Ω is the bounded region between $y^2 = 2x$ and $y^2 = 8 - 2x$.

(d) $\iint\limits_{\Omega} e^{-y^2/2}\, dxdy,$ Ω is the triangular region bounded by the y-axis, $2y = x$, $y = 1$.

(e) $\iint\limits_{\Omega} (3xy^3 - y)\, dxdy,$ Ω is the region between $y = |x|$ and $y = -|x|$, $x \in [-1, 1]$.