

Calculus — Homework 3 (Spring 2025)

1. Evaluate the integrals.

(a) $\int_0^1 \frac{dx}{\sqrt{x}}.$

(c) $\int_0^1 x \ln x \, dx.$

(b) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}.$

2. Does the integral converge or diverge?

(a) $\int_{1/2}^2 \frac{dx}{x \ln x}.$

(d) $\int_{-\infty}^1 \frac{\sqrt{x+1}}{x^2} \, dx.$

(b) $\int_0^{\pi/2} \tan x \, dx.$

(e) $\int_{\pi}^{\infty} \frac{2 + \cos x}{x} \, dx.$

(c) $\int_1^{\infty} \frac{dx}{x^3 + 1}.$

(f) $\int_1^{\infty} \frac{e^x}{x} \, dx.$

3. Determine whether the series converges or diverges. If it converges, find the sum of the series. If it diverges, explain why.

(a) $\sum_{k=3}^{\infty} \frac{1}{k^2 - k}.$

(c) $\sum_{k=0}^{\infty} \frac{3^{k-1}}{4^{3k+1}}.$

(b) $\sum_{k=0}^{\infty} \frac{(-1)^k}{5^k}.$

(d) $\sum_{k=1}^{\infty} \left(\frac{k-1}{k}\right)^k.$

4. Let j be a positive integer.

(a) Show that

$$\sum_{k=0}^{\infty} a_k \text{ converges} \quad \text{iff} \quad \sum_{k=j}^{\infty} a_k \text{ converges}.$$

(b) Show that if $\sum_{k=0}^{\infty} a_k = L$, then $\sum_{k=j}^{\infty} a_k = L - \sum_{k=0}^{j-1} a_k.$

(c) Show that if $\sum_{k=j}^{\infty} a_k = M$, then $\sum_{k=0}^{\infty} a_k = M + \sum_{k=0}^{j-1} a_k.$

5. Determine whether the series converges or diverges. Explain why.

(a) $\sum_{k=2}^{\infty} \frac{k}{k^3 - k}.$

(e) $\sum_{k=1}^{\infty} k^2 2^{-k^3}.$

(i) $\sum_{k=1}^{\infty} \frac{k^2}{e^k}.$

(b) $\sum_{k=1}^{\infty} \frac{\ln k}{k}.$

(f) $\sum_{k=1}^{\infty} \frac{2 + \cos k}{\sqrt{k+1}}.$

(j) $\sum_{k=1}^{\infty} \frac{k^k}{3^{k^2}}.$

(c) $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}.$

(g) $\sum_{k=1}^{\infty} \frac{10^k}{k!}.$

(k) $\sum_{k=1}^{\infty} \frac{2 \cdot 4 \cdots 2k}{(2k)!}.$

(d) $\sum_{k=1}^{\infty} \frac{1}{1 + 2 \ln k}.$

(h) $\sum_{k=1}^{\infty} \frac{1}{k 2^k}.$

(l) $\sum_{k=1}^{\infty} \frac{k!}{k^{k/2}}.$