Calculus (I) — Homework 8 (Fall 2024)

1. Given that
$$\int_0^1 f(x)\,dx = 6, \qquad \int_0^2 f(x)\,dx = 4, \qquad \int_1^2 g(x)\,dx = 1,$$
find the following:

(a) $\int_1^2 f(x)\,dx.$

(b) $\int_0^0 f(x)\,dx.$

(c) $\int_2^0 f(x)\,dx.$

(d) $\int_1^2 (f(x) + g(x))\,dx.$

(e) $\int_2^1 (3f(x) - 2g(x))\,dx.$

2. Show that
$$\frac{1}{2} \le \int_1^2 \frac{1}{x}\,dx \le 1.$$

3. Find the critical point(s) for
$$F(x) = \int_0^x \frac{t-1}{1+t^2}\,dt.$$
At each critical point, determine whether F has a local maximum, a local minimum, or neither.

4. Calculate $F'(x)$.

(a) $F(x) = \int_0^{x^3} t\cos t\,dt.$

(b) $F(x) = \int_{x^2}^1 (t - \sin^2 t)\,dt.$

5. (A mean value theorem for integrals.) Show that if f is continuous on $[a, b]$, then there is at least one number c in (a, b) for which
$$\int_a^b f(x)\,dx = f(c)\cdot(b-a).$$

6. Evaluate the integral.

(a) $\int_0^1 (2x-3)\,dx.$

(b) $\int_1^2 (2x+x^2)\,dx.$

(c) $\int_1^4 2\sqrt{x}\,dx.$

(d) $\int_0^1 (x^{3/4} - 2x^{1/2})\,dx.$

(e) $\int_0^1 (x+1)^{17}\,dx.$

(f) $\int_1^2 \frac{6-x}{x^3}\,dx.$

(g) $\int_1^2 2x(x^2+1)\,dx.$

(h) $\int_0^{\pi/2} \cos x\,dx.$

(i) $\int_0^{2\pi} \sin x\,dx.$

(j) $\int_{-\pi/6}^{\pi/6} \sin x\cos x\,dx.$

(k) $\int_2^5 (x-3)\,dx.$

(l) $\int_2^5 |x-3|\,dx.$