Calculus (I) — Homework 7 (Fall 2024)

1. Find the critical points, local maximums and local minimums of f.

   (a) $f(x) = x^3 - 3x + 2$.

   (b) $f(x) = x + \frac{1}{x}$.

   (c) $f(x) = |x^2 - 5|$.

   (d) $f(x) = x - \cos x$.

2. True or false? Explain your answers.

   (a) The equation $6x^4 - 7x + 1 = 0$ has 4 distinct real roots.

   (b) The equation $x^5 + 13x + 1 = 0$ has exactly one real root.

3. Find the critical points. Then find and classify all the extreme values.

   (a) $f(x) = x^2 - 4x + 1, \quad 0 \le x \le 3$.

   (b) $f(x) = \dfrac{x^2}{1 + x^2}, \quad -1 \le x \le 2$.

   (c) $f(x) = \sin 2x - x, \quad 0 \le x \le \pi$.

   (d) $f(x) = 1 - \sqrt[3]{x - 1}, \quad x \in (-\infty, \infty)$.

   (e) $f(x) = \begin{cases} x^2 + 2x + 2, & x < 0, \\ x^2 - 2x + 2, & 0 \le x \le 2. \end{cases}$

4. Find the greatest possible value of xy given that x and y are both positive and $x + y = 40$.

5. Describe the concavity of the graph and find the points of inflection (if any).

   (a) $f(x) = x + \dfrac{1}{x}$.

   (b) $f(x) = x^3(1 - x)$.

   (c) $f(x) = \sin^2 x, \quad 0 < x < \pi$.

6. Determine A and B so that the curve

$$y = A\cos 2x + B\sin 3x$$

   has a point of inflection at $(1, 4)$.

7. Calculate the following limits.

   (a) $\displaystyle\lim_{x \to 0^+} \frac{\sin x}{\sqrt{x}}$.

   (b) $\displaystyle\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$.

   (c) $\displaystyle\lim_{x \to 0} \frac{x + \sin(\pi x)}{x - \sin(\pi x)}$.

   (d) $\displaystyle\lim_{x \to 0} \frac{\cos x - \cos 3x}{\sin(x^2)}$.

   (e) $\displaystyle\lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2 - x}}{x}$.

   (f) $\displaystyle\lim_{x \to 1} \frac{x^{1/2} - x^{1/4}}{x - 1}$.

   (g) $\displaystyle\lim_{x \to 0} \frac{1}{x^2} - \frac{1}{\sin^2 x}$.

   (h) $\displaystyle\lim_{x \to \frac{\pi}{2}} (x - \frac{\pi}{2}) \sec x$.

8. Read the descriptions about upper (Riemann) sum and lower (Riemann) sum on the following websites:

   - upper Riemann sum
   - lower Riemann sum