Calculus (I) — Homework 5 (Fall 2024)

- 1. Differentiate the following functions.
 - (a) $f(x) = 3\cos x 4\sec x$. (b) $f(x) = \sin^2 x$. (c) $f(x) = \tan(x^2)$. (d) $f(x) = \cos(\sqrt{x}), x > 0$. (e) $f(x) = \sqrt{1+x^2}, x > 0$. (f) $f(x) = \sqrt{\sin x \cos x}, 0 < x < \pi/2$. (g) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}, x > 0$. (h) $f(x) = (x + 1)^{1/3}(x + 2)^{2/3}$.
- 2. Express dy/dx in terms of x and y.

(a)
$$x^2 + y^2 = 4$$
.
(b) $x^3 + y^3 - 3xy = 0$.
(c) $\sin(x + y) = xy$.
(d) $\sqrt{x} + \sqrt{y} = 4$, $x, y > 0$.

- 3. Find equations for the tangent line at the point indicated.
 - (a) $9x^2 + 4y^2 = 72$; (2,3).
 - (b) $x^2 + xy + 2y^2 = 28$; (-2, -3).
 - (c) $x = \cos y$; $(\frac{1}{2}, \frac{\pi}{3})$.
- 4. Find dy/dx, d^2y/dx^2 , \cdots , d^8y/dx^8 .
 - (a) $y = \sin x$.
 - (b) $y = \cos x$.
- 5. Find the rate of change of the volume of a cube with respect to the length *s* of a side. What is the rate when s = 4?
- 6. Let

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

- Is f differentiable at x = 0? Is f twice differentiable (i.e. f' is differentiable) at x = 0? Prove your answers.
- 7. Find the smallest positive integer *n* such that $\frac{d^n}{dx^n}(x^{10}\sin x)\Big|_{x=0} \neq 0$ and find this value. (Hint: derive a formula of $\frac{d^n}{dx^n}(fg)$ from the product rule.)