

Calculus (I) — Homework 4 (Fall 2024)

1. Write an equation for the tangent line at $(c, f(c))$.

(a) $f(x) = x^2 - 4x$, $c = 3$.

(b) $f(x) = \sqrt{x}$, $c = 1$.

2. Draw the graph of f ; indicate where f is not differentiable, and indicate where f is not continuous.

(a) $f(x) = \sqrt{|x|}$.

(b) $f(x) = |x^2 - 4|$.

(c) $f(x) = \begin{cases} x^2, & |x| \leq 1, \\ 2 - x, & |x| > 1. \end{cases}$

3. Differentiate the following functions.

(a) $f(x) = 1 - x$.

(d) $f(x) = (x^2 - 1)(x - 3)$.

(g) $f(x) = (1 - 2x)^{-1}$.

(b) $f(x) = 11x^5 - 6x^3 + 8$.

(e) $f(x) = \frac{x^3}{1 - x}$.

(h) $f(x) = (1 + 2x)^5$.

(c) $f(x) = \frac{3}{x^2}$.

(f) $f(x) = \left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$.

(i) $f(x) = \left(x - \frac{1}{x}\right)^4$.

4. Find the point(s) where the tangent line is horizontal.

(a) $f(x) = (x - 2)(x^2 - x - 11)$.

(b) $f(x) = x^2 - \frac{16}{x}$.

5. Find dy/dx , d^2y/dx^2 and d^3y/dx^3 .

(a) $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + 1$.

(c) $y = x^3 - \frac{1}{x^3}$.

(b) $y = (1 + 5x)^2$.

(d) $y = \frac{x^4 + 2}{x}$.

6. Let p be an arbitrary polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0, \quad a_n \neq 0.$$

(a) Find $(d^{n-1}/dx^{n-1})(p(x))$ and $(d^n/dx^n)(p(x))$.

(b) What is $(d^k/dx^k)(p(x))$ for $k > n$?

7. Let f, g be arbitrary differentiable functions.

(a) Find a formula for $(f \cdot g)''(x)$.

(b) Assume $g(x) \neq 0$. Find a formula for $(f/g)''(x)$.

8. True or false? Explain your answers.

(a) The derivative of f at c is the limit $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$.

(b) If a function f is differentiable at c , then f is continuous at c .

(c) If a function f is continuous at c , then f is differentiable at c .