

Calculus (I) — Homework 2 (Fall 2024)

1. True or false? Explain your answers.

- (a) If $\lim_{x \rightarrow c} (f(x) + g(x))$ exists but $\lim_{x \rightarrow c} f(x)$ does not exist, then $\lim_{x \rightarrow c} g(x)$ does not exist.
- (b) If $\lim_{x \rightarrow c} (f(x) + g(x))$ and $\lim_{x \rightarrow c} f(x)$ exist, then $\lim_{x \rightarrow c} g(x)$ exists.
- (c) If $\lim_{x \rightarrow c} \sqrt{f(x)}$ exists, then $\lim_{x \rightarrow c} f(x)$ exists.
- (d) If $\lim_{x \rightarrow c} f(x)$ exists, then $\lim_{x \rightarrow c} \sqrt{f(x)}$ exists.
- (e) If $\lim_{x \rightarrow c} f(x)$ exists, then $\lim_{x \rightarrow c} \frac{1}{f(x)}$ exists.
- (f) If $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ exists and $\lim_{x \rightarrow c} g(x) = 0$, then $\lim_{x \rightarrow c} f(x) = 0$.

2. Evaluate the limits that do exist. If the limit does not exist, explain why.

- (a) $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$.
- (b) $\lim_{x \rightarrow 0} \frac{3x}{\sin 5x}$.
- (c) $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 5x}$.
- (d) $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x}$.
- (e) $\lim_{x \rightarrow 0} \frac{\sin^2(x^2)}{x^2}$.
- (f) $\lim_{x \rightarrow 0} \frac{\tan^2(3x)}{4x^2}$.
- (g) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{2x}$.
- (h) $\lim_{x \rightarrow \pi/4} \frac{\sin x}{x}$.
- (i) $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$.

3. State the precise definition of $\lim_{x \rightarrow -\infty} f(x) = L$, and use it to prove that $\lim_{x \rightarrow -\infty} \frac{1}{x} = 0$.

4. Evaluate the limit $\lim_{x \rightarrow 0} |x| \cos\left(\frac{1}{x^2}\right)$. Prove your answer.

5. Evaluate the limits.

- (a) $\lim_{x \rightarrow \infty} \sqrt{\frac{8x^2 - 3}{2x^2 + x}}$.
- (b) $\lim_{x \rightarrow -\infty} \left(\frac{1 - x^3}{x^2 + 7x}\right)^5$.
- (c) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x + 1}$.
- (d) $\lim_{x \rightarrow \infty} \frac{2x^{5/3} - x^{1/3} + 7}{x^{9/5} + 3x + \sqrt{x}}$.