Calculus (I) — Homework 1 (Fall 2024)

1. State the precise definition of $\lim_{x\to c} f(x) = L$ and use it to show that

$$\lim_{x\to 3}(x-1)^2 = 4.$$

2. Use the ε-δ argument (i.e. the precise definition of limit) to prove that: if $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = M$ and $\lim_{x\to c} h(x) = N$, then

$$\lim_{x\to c}\Big(3f(x) + 4g(x) - 5h(x)\Big) = 3L + 4M - 5N.$$

3. Decide whether or not the indicated limit exists. Evaluate the limits that do exist. If the limit does not exist, explain why.

(a) $\lim_{x\to -1} |x|(x^4 - 3)$.

(b) $\lim_{x\to 1} \dfrac{x}{x+1}$.

(c) $\lim_{x\to -1} \dfrac{1-x}{x+1}$.

(d) $\lim_{x\to 0} \dfrac{x(x+1)}{2x^2}$.

(e) $\lim_{x\to 1} \dfrac{x}{|x|}$.

(f) $\lim_{x\to 1^+} \dfrac{\sqrt{x-1}}{x}$.

(g) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$.

(h) $\lim_{x\to -1^+} x^3(x^4+1)$.

(i) $\lim_{x\to 1} \dfrac{x^{10}-1}{x^3-1}$.

(j) $\lim_{x\to -4} \Big(\dfrac{2x}{x+4} + \dfrac{8}{x+4}\Big)$.

(k) $\lim_{h\to 0} h\Big(1 - \dfrac{1}{h}\Big)$.

(l) $\lim_{h\to 0} \dfrac{1 - 1/h^2}{1 + 1/h^2}$.

(m) $\lim_{x\to 2^+} f(x)$ if $f(x) = \begin{cases} 2x-1, & x \le 2 \\ x^2 - x, & x > 2. \end{cases}$

(n) $\lim_{x\to -1^-} f(x)$ if $f(x) = \begin{cases} 1, & x \le -1 \\ x+2, & x > -1. \end{cases}$

(o) $\lim_{x\to 2} f(x)$ if $f(x) = \begin{cases} 3, & x \text{ an integer} \\ x+2, & \text{otherwise.} \end{cases}$

4. Given that

$$\lim_{x\to c} f(x) = 2, \qquad \lim_{x\to c} g(x) = -1, \qquad \lim_{x\to c} h(x) = 0,$$

evaluate the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to c}\Big(f(x) - g(x)\Big)$.

(b) $\lim_{x\to c}\Big(f(x) + 3g(x)\Big)^3$.

(c) $\lim_{x\to c}\Big(f(x)g(x)h(x)\Big)$.

(d) $\lim_{x\to c} \dfrac{f(x)}{g(x)}$.

(e) $\lim_{x\to c} \dfrac{g(x)}{h(x)}$.

(f) $\lim_{x\to c} \dfrac{f(x)g(x)}{f(x)+g(x)}$.