

Calculus (I) — Homework 12 (Fall 2024)

1. Calculate.

(a) $\int \frac{1}{x^2 + 7x + 6} dx.$

(b) $\int \frac{x^5}{(x-2)^2} dx.$

(c) $\int \frac{1}{x^2 - 6x + 13} dx.$

(d) $\int \frac{x}{x^2 - 6x + 13} dx.$

(e) $\int \frac{1}{(x^2 - 2x + 2)^2} dx.$

(f) $\int \frac{x}{(x^2 - 2x + 2)^2} dx.$

(g) $\int \frac{x}{x^4 - 1} dx.$

(h) $\int \frac{x^5 + x^2}{x^4 - 1} dx.$

(i) $\int \frac{x^4}{x^3 + x^2 - 2x} dx.$

(j) $\int \frac{1}{(x-1)(x^2+1)^2} dx.$

2. Sketch the region bounded by the curves and calculate the area of the region.

(a) $4x = 4y - y^2, \quad 4x - y = 0.$

(b) $x = y^2, \quad x = 12 - 2y^2.$

3. Find the length of the following curves.

(a) $y = \frac{x^3}{12} + \frac{1}{x}, \quad 1 \leq x \leq 4.$

(b) $x = 2y^{3/2}, \quad 0 \leq y \leq 1.$

4. Find the area of the surface generate by revolving the curve $y = 2\sqrt{x}, 1 \leq x \leq 2$, about the x -axis.

5. Sketch the region Ω bounded by the curves and find the volume of the solid generated by revolving this region about the x -axis.

(a) $y = x, \quad y = 0, \quad x = 1.$

(b) $y = \cos x, \quad y = x + 1, \quad x = \frac{1}{2}\pi.$

6. Sketch the region Ω bounded by the curves and find the volume of the solid generated by revolving this region about the y -axis.

(a) $y = x, \quad y = 0, \quad x = 1.$

(b) $y = x^2, \quad y = x^{1/3}.$

7. Determine whether the functions satisfy the differential equation.

(a) $y' + xy = x; \quad y_1(x) = e^{-x^2/2}, \quad y_2(x) = 1 + e^{-x^2/2}.$

(b) $y'' + 4y = 0; \quad y_1(x) = 2 \sin 2x, \quad y_2(x) = 2 \cos x, \quad y_3(x) = \sinh 2x.$

8. Find the function $f(x)$ which satisfies the equations

$$f'(x) = 2f(x), \quad f(1) = e^2.$$

Find approximations of the values $f(10), f(20)$ and $f(30)$.

9. Solve the differential equation.

(a) $2\sqrt{xy} \frac{dy}{dx} = 1, \quad x, y > 0.$

(b) $\frac{dy}{dx} = e^{x-y}.$

(c) $y^2 \frac{dy}{dx} = 3x^2y^3 - 6x^2.$