

Calculus (I) — Homework 11 (Fall 2024)

1. Calculate.

(a) $\int_0^1 x e^{-x} dx.$

(b) $\int_0^1 x^3 e^{-x^2} dx.$

(c) $\int_1^{e^2} x \ln(\sqrt{x}) dx.$

(d) $\int_0^{1/2} x \cos \pi x dx.$

(e) $\int_0^{1/4} \arcsin 2x dx.$

(f) $\int x^2(e^x - 1) dx.$

(g) $\int (\ln x)^2 dx.$

(h) $\int e^x \sin x dx.$

2. Calculate.

(a) $\int_0^{\pi/6} \sin^2 3x dx.$

(b) $\int_0^{\pi} \sin^3 x dx.$

(c) $\int \sin^3 x \cos^3 x dx.$

(d) $\int \sin^2 x \cos^4 x dx.$

3. Use integration by parts to show that for $n > 2$,

$$\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx.$$

Then compute

$$\int_0^{\pi/2} \sin^{2k} x dx \quad \text{and} \quad \int_0^{\pi/2} \sin^{2k+1} x dx$$

for $k \geq 1$.

4. Calculate.

(a) $\int_0^2 \frac{x^2}{\sqrt{16-x^2}} dx.$

(b) $\int \frac{x^2}{\sqrt{4-x^2}} dx.$

(c) $\int \frac{x^2}{\sqrt{x^2-4}} dx.$

(d) $\int \frac{x^2}{\sqrt{4+x^2}} dx.$

(e) $\int \frac{x}{\sqrt{x^2-2x+3}} dx.$

(f) $\int \frac{1}{(x^2+1)^3} dx.$

5. Decompose the rational functions into the form

$$p(x) + \frac{q_1(x)}{(x-b_1)^{r_1}} + \cdots + \frac{q_i(x)}{(x-b_i)^{r_i}} + \frac{q_{i+1}(x)}{(x^2+c_{i+1}x+d_{i+1})^{r_{i+1}}} + \cdots + \frac{q_j(x)}{(x^2+c_jx+d_j)^{r_j}},$$

where $p(x)$, $q_k(x)$ are polynomials, and the degree of $q_k(x)$ is smaller than the degree of denominator.

(a) $\frac{1}{x^2+7x+6}.$

(b) $\frac{x}{x^4-1}.$

(c) $\frac{x^5+x^2}{x^4-1}.$

(d) $\frac{x^4}{x^3+x^2-2x}.$