Calculus (I) — Homework 10 (Fall 2024)

1. Differentiate.

   (a) $f(x) = e^x \ln(x^2+1)$.

   (b) $f(x) = e^{\sin 2x}$.

   (c) $f(x) = \ln(2 + \cos e^{2x})$.

   (d) $f(x) = \dfrac{e^{2x}-1}{e^{2x}+1}$.

   (e) $f(x) = \sqrt{\log_3 x}$.

   (f) $f(x) = x^x$.

   (g) $f(x) = (\ln x)^x$.

   (h) $f(x) = \cos(2^x)$.

2. Evaluate.

   (a) $\displaystyle\int_0^1 \frac{4-e^x}{e^x}\,dx$.

   (b) $\displaystyle\int_0^1 \frac{e^x}{4-e^x}\,dx$.

   (c) $\displaystyle\int_0^1 x(e^{x^2}+2)\,dx$.

   (d) $\displaystyle\int_0^1 \frac{4}{\sqrt{e^x}}\,dx$.

   (e) $\displaystyle\int_1^2 2^{-x}\,dx$.

   (f) $\displaystyle\int_1^2 x10^{1+x^2}\,dx$.

   (g) $\displaystyle\int_1^4 \frac{dx}{x\ln 2}$.

   (h) $\displaystyle\int_1^5 \frac{\log_5 x}{x}\,dx$.

3. Find the number(s) x which satisfies the equation.

   (a) $10 = e^x$.

   (b) $\log_x 2 = \log_3 x$.

4. Evaluate.

   (a) $\displaystyle\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^x$.

   (b) $\displaystyle\lim_{x\to 0^+} x^x$.

5. Prove that, if n is a positive integer, then there exists N such that

   $$e^x > x^n$$

   for all $x \geq N$. (This is a famous property. You can find a proof online.)

6. Let

   $$f(x) = \begin{cases} e^{-1/x^2}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

   Is f differentiable at $x = 0$? Is f twice differentiable at $x = 0$? Justify your answers.

7. Determine the exact value.

   (a) $\arcsin(-\sqrt{3}/2)$.

   (b) $\cos(\arctan 2)$.

   (c) $\arcsin\left(\sin(11\pi/6)\right)$.

   (d) $\cos\left(2\arcsin(4/5)\right)$.

8. Differentiate.

   (a) $f(x) = \arctan(x+1)$.

   (b) $f(x) = e^x \arcsin x$.

9. Evaluate.

(a) $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx.$

(b) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{4-x^2}}\,dx.$

(c) $\displaystyle\int_{0}^{3/2} \frac{dx}{9+4x^2}.$

(d) $\displaystyle\int_{\ln 2}^{\ln 3} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx.$

(e) $\displaystyle\int_{0}^{\ln 2} \sinh 2x\,dx.$

(f) $\displaystyle\int_{0}^{\ln 2} x\sinh x\,dx.$

(g) $\displaystyle\int_{0}^{1} \frac{dx}{5^x}.$

(h) $\displaystyle\int_{0}^{1} \frac{x^3}{1+x^4}\,dx.$

(i) $\displaystyle\int_{1}^{2} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx.$

(j) $\displaystyle\int_{-3}^{-2} \frac{dx}{x^2+6x+10}.$