Calculus — Homework 10 (Spring 2024)

1. Find maxima or minima with side conditions.
   (a) Minimize $x^2 + y^2$; on the hyperbola $xy = 1$.
   (b) Maximize $x + y$ on the curve $x^4 + y^4 = 1$.
   (c) Minimize $x + 2y + 4z$ on the sphere $x^2 + y^2 + z^2 = 7$.
   (If you are not familiar with functions of three variables, you can find examples on page 846 in the textbook.)

2. Let x, y, z be the three angles of a triangle. Determine the maximum value of

$$f(x, y, z) = \sin x \sin y \sin z.$$

3. Let R be the rectangle $a \le x \le b$, $c \le y \le d$. Show that, if f is continuous on $[a, b]$ and g is continuous on $[c, d]$, then

$$\iint_R f(x)g(y)\,dxdy = \left(\int_a^b f(x)\,dx\right) \cdot \left(\int_c^d g(y)\,dy\right).$$

4. Evaluate the integral for $\Omega : 0 \le x \le 1, 0 \le y \le 3$.

   (a) $\displaystyle\iint_\Omega x^2\,dxdy.$ (b) $\displaystyle\iint_\Omega e^{x+y}\,dxdy.$

5. Evaluate the integral for $\Omega : 0 \le x \le 1, 0 \le y \le x$.

   (a) $\displaystyle\iint_\Omega x^3 y\,dxdy.$ (b) $\displaystyle\iint_\Omega x^2 y^2\,dxdy.$

6. Evaluate the double integral.
   (a) $\displaystyle\iint_\Omega (x + 3y^3)\,dxdy,\quad \Omega : 0 \le x^2 + y^2 \le 1.$
   (b) $\displaystyle\iint_\Omega \sqrt{xy}\,dxdy,\quad \Omega : 0 \le y \le 1, y^2 \le x \le y.$
   (c) $\displaystyle\iint_\Omega (4 - y^2)\,dxdy,\quad \Omega$ is the bounded region between $y^2 = 2x$ and $y^2 = 8 - 2x$.
   (d) $\displaystyle\iint_\Omega e^{-y^2/2}\,dxdy,\quad \Omega$ is the triangular region bounded by the y-axis, $2y = x$, $y = 1$.
   (e) $\displaystyle\iint_\Omega (3xy^3 - y)\,dxdy,\quad \Omega$ is the region between $y = |x|$ and $y = -|x|$, $x \in [-1, 1]$.

7. Find the area of the first quadrant region bounded by $xy = 2$, $y = 1$, $y = x + 1$.

8. Find the volume under the paraboloid $z = x^2 + y^2$ within the cylinder $x^2 + y^2 \le 1$, $z \ge 0$.