Calculus — Homework 9 (Fall 2023)

1. Sketch the region bounded by the curves and calculate the area of the region.
   (a) $4x = 4y - y^2, \qquad 4x - y = 0.$
   (b) $x = y^2, \qquad x = 12 - 2y^2.$

2. Find the length of the following curves.
   (a) $y = \dfrac{x^3}{12} + \dfrac{1}{x}, \qquad 1 \le x \le 4.$
   (b) $x = 2y^{3/2}, \qquad 0 \le y \le 1.$

3. Find the area of the surface generate by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis.

4. Sketch the region Ω bounded by the curves and find the volume of the solid generated by revolving this region about the x-axis.
   (a) $y = x, \qquad y = 0, \qquad x = 1.$
   (b) $y = x^3, \qquad x + y = 10, \qquad y = 1.$
   (c) $y = \cos x, \qquad y = x + 1, \qquad x = \frac{1}{2}\pi.$

5. Sketch the region Ω bounded by the curves and find the volume of the solid generated by revolving this region about the y-axis.
   (a) $y = x, \qquad y = 0, \qquad x = 1.$
   (b) $y = x^2, \qquad y = x^{1/3}.$

We choose the codomain to be the range of a function in the following questions.

6. Determine whether or not the function is one-to-one. If the function has an inverse, find it and sketch the graphs of the function and its inverse.
   (a) $f(x) = 5x + 3, \quad x \in (-\infty, \infty).$
   (b) $f(x) = 1 - x^2, \quad x \in (-\infty, \infty).$
   (c) $f(x) = \sin x, \quad x \in [-\frac{\pi}{2}, \frac{\pi}{2}].$
   (d) $f(x) = \cos x, \quad x \in [-\frac{\pi}{2}, \frac{\pi}{2}].$

7. Verify that f has an inverse and find $(f^{-1})'(c)$.
   (a) $f(x) = x^3 + 1$, $x \in (-\infty, \infty)$; $c = 9.$
   (b) $f(x) = 1 - 2x - x^3$, $x \in (-\infty, \infty)$; $c = 4.$
   (c) $f(x) = \sin x$, $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$; $c = -\frac{1}{2}.$
   (d) $f(x) = \dfrac{x+3}{x-1}$, $x > 1$; $c = 3.$
   (e) $f(x) = \int_2^x \sqrt{1+t^2}\, dt$, $x \in (-\infty, \infty)$; $c = 0.$

8. Set
$$f(x) = \int_2^x \sqrt{1+t^2}\, dt.$$
   (a) Show that f has an inverse.
   (b) Find $(f^{-1})'(0)$.