

## Calculus — Homework 9.5 (Fall 2023)

1. Estimate the values.

(a)  $\ln 20$ .

(b)  $\ln 16$ .

(c)  $\ln 3^4$ .

( $\ln 2 \approx 0.7$ ,  $\ln 3 \approx 1.1$ ,  $\ln 5 \approx 1.6$ .)

**Solution:**

(a)  $\ln 20 \approx 3$

(b)  $\ln 16 \approx 2.8$

(c)  $\ln 3^4 \approx 4.4$

2. Show that

$$\lim_{x \rightarrow 1} \frac{\ln x}{x - 1} = 1.$$

(Hint: Note that  $\frac{\ln x}{x-1} = \frac{\ln x - \ln 1}{x-1}$  and interpret the limit as a derivative.)

3. Determine the maximal possible domain and find the derivative.

(a)  $f(x) = \ln 4x$ .

(b)  $f(x) = (\ln x)^3$ .

(c)  $f(x) = \ln(\ln x)$ .

(d)  $f(x) = \frac{1}{\ln x}$ .

(e)  $f(x) = \ln \left| \frac{x+2}{x^3-1} \right|$ .

(f)  $f(x) = \sin(\ln x)$ .

**Solution:**

(a) Domain :  $\{x > 0\}$  , Derivative :  $\frac{1}{x}$

(b) Domain :  $\{x > 0\}$  , Derivative :  $3(\ln x)^2 \frac{1}{x}$

(c) Domain :  $\{x > 1\}$  , Derivative :  $\frac{1}{x \ln x}$

(d) Domain :  $(0, 1) \cup (1, \infty)$  , Derivative :  $-\frac{1}{x(\ln x)^2}$

(e) Domain :  $\{x \neq 1, x \neq -2\}$  , Derivative :  $\frac{-2x^3-6x^2-1}{(x+2)(x^3-1)}$

(f) Domain :  $\{x > 0\}$  , Derivative :  $\cos(\ln x) \frac{1}{x}$

4. Show that  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$ . (Hint: First show the inequality  $\frac{1}{x+1} < \ln\left(1 + \frac{1}{x}\right) < \frac{1}{x}$  by noting  $1 > \frac{1}{t} > \frac{x}{1+x}$  for  $t \in (1, 1 + \frac{1}{x})$ .)

5. Evaluate.

(a)  $\int_1^e \frac{dx}{x}$ .

(b)  $\int_e^{e^2} \frac{dx}{x}$ .

(c)  $\int_4^5 \frac{x}{x^2-1} dx$ .

(d)  $\int_{1/4}^{1/3} \tan(\pi x) dx$ .

(e)  $\int_{\pi/6}^{\pi/2} \frac{\cos x}{1 + \sin x} dx$ .

(f)  $\int_1^e \frac{\ln x}{x} dx$ .

$$(g) \int_1^2 \frac{1}{\sqrt{x}(1+\sqrt{x})} dx.$$

$$(h) \int_0^{\pi/2} \frac{\sin x - \cos x}{\sin x + \cos x} dx.$$

**Solution:**

(a) 1	(c) $\frac{1}{2} \ln \frac{8}{5}$	(e) $\ln \frac{4}{3}$	(g) $2 \ln(\frac{1+\sqrt{2}}{2})$
(b) 1	(d) $\frac{1}{2\pi} \ln 2$	(f) $\frac{1}{2}$	(h) 0

6. Differentiate.

(a) $y = e^x$ .	(c) $y = e^x \ln(x^2 + 1)$ .	(e) $y = \ln(2 + \cos e^{2x})$ .
(b) $y = e^{x^2-1}$ .	(d) $y = e^{\sin 2x}$ .	(f) $y = \frac{e^{2x} - 1}{e^{2x} + 1}$ .

**Solution:**

(a) $y' = e^x$	(d) $y' = 2e^{\sin 2x} \cos 2x$
(b) $y' = 2xe^{x^2-1}$	(e) $y' = -2e^{2x} \frac{\sin e^{2x}}{2 + \cos e^{2x}}$
(c) $y' = e^x \ln(x^2 + 1) + e^x \frac{2x}{x^2+1}$	(f) $y' = \frac{4e^{2x}}{(e^{2x}+1)^2}$

7. Evaluate.

(a) $\int_0^1 e^x dx$ .	(c) $\int_0^1 \frac{4 - e^x}{e^x} dx$ .	(e) $\int_0^1 x(e^{x^2} + 2) dx$ .
(b) $\int_0^{\ln \pi} e^{-6x} dx$ .	(d) $\int_0^1 \frac{e^x}{4 - e^x} dx$ .	(f) $\int_0^1 \frac{4}{\sqrt{e^x}} dx$ .

**Solution:**

(a) $e - 1$	(c) $-\frac{4}{e} + 3$	(e) $\frac{e+1}{2}$
(b) $\frac{1}{6}(1 - \frac{1}{\pi^6})$	(d) $-\ln(4 - e) + \ln 3$	(f) $-8e^{-\frac{1}{2}} + 8$

8. Differentiate.

(a) $f(x) = 3^{2x}$ .	(d) $f(x) = (\ln x)^x$ .
(b) $f(x) = 2^{5x} 3^{\ln x}$ .	(e) $f(x) = \cos(2^x)$ .
(c) $f(x) = x^x$ .	

**Solution:**

(a) $2 \ln 3 \cdot 3^{2x}$	(d) $(\ln(\ln x) + \frac{1}{\ln x})(\ln x)^x$
(b) $(5 \ln 2 + \frac{\ln 3}{x}) \cdot 2^{5x} 3^{\ln x}$	
(c) $(1 + \ln x)x^x$	(e) $-\sin(2^x) \cdot 2^x \ln 2$

9. Evaluate.

$$(a) \int_1^2 2^{-x} dx.$$

$$(b) \int_1^2 x 10^{1+x^2} dx.$$

$$(c) \int_1^4 \frac{dx}{x \ln 2}.$$

**Solution:**

$$(a) \frac{1}{4 \ln 2}$$

$$(b) \frac{10^5 - 10^2}{2 \ln 10}$$

$$(c) 2$$