Calculus — Homework 7 (Fall 2023)

1. Given that
$$\int_0^1 f(x)\,dx = 6, \qquad \int_0^2 f(x)\,dx = 4, \qquad \int_1^2 g(x)\,dx = 1,$$
find the following:

   (a) $\displaystyle\int_1^2 f(x)\,dx.$

   (b) $\displaystyle\int_0^0 f(x)\,dx.$

   (c) $\displaystyle\int_2^0 f(x)\,dx.$

   (d) $\displaystyle\int_1^2 \big(f(x)+g(x)\big)\,dx.$

   (e) $\displaystyle\int_2^1 \big(3f(x)-2g(x)\big)\,dx.$

2. Show that
$$\frac{1}{2} \le \int_1^2 \frac{1}{x}\,dx \le 1.$$

3. Find the critical point(s) for
$$F(x) = \int_0^x \frac{t-1}{1+t^2}\,dt.$$
At each critical point, determine whether F has a local maximum, a local minimum, or neither.

4. Calculate $F'(x)$.

   (a) $F(x) = \displaystyle\int_0^{x^3} t\cos t\,dt.$

   (b) $F(x) = \displaystyle\int_{x^2}^1 (t-\sin^2 t)\,dt.$

5. (A mean value theorem for integrals.) Show that if f is continuous on $[a,b]$, then there is at least one number c in (a,b) for which
$$\int_a^b f(x)\,dx = f(c)\cdot(b-a).$$