

## Calculus — Homework 5 (Fall 2023)

1. Differentiate the following functions.

(a)  $f(x) = (1 - 2x)^{-1}$ .

(b)  $f(x) = (1 + 2x)^5$ .

(c)  $f(x) = \left(x - \frac{1}{x}\right)^4$ .

(d)  $f(x) = 3 \cos x - 4 \sec x$ .

(e)  $f(x) = \sin^2 x$ .

(f)  $f(x) = \tan(x^2)$ .

(g)  $f(x) = \cos(\sqrt{x})$ ,  $x > 0$ .

(h)  $f(x) = \sqrt[3]{\frac{x}{1+x^2}}$ ,  $x > 0$ .

(i)  $f(x) = \sqrt{\sin x \cos x}$ ,  $0 < x < \pi/2$ .

(j)  $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ ,  $x > 0$ .

(k)  $f(x) = (x + 1)^{1/3}(x + 2)^{2/3}$ .

2. Find  $dy/dx$ ,  $d^2y/dx^2$ ,  $\dots$ ,  $d^8y/dx^8$ .

(a)  $y = \sin x$ .

(b)  $y = \cos x$ .

3. Let

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Is  $f$  differentiable at  $x = 0$ ? Is  $f$  twice differentiable (i.e.  $f'$  is differentiable) at  $x = 0$ ? Explain your answers.

4. Find the smallest positive integer  $n$  such that  $\frac{d^n}{dx^n}(x^{10} \sin x)\Big|_{x=0} \neq 0$  and find this value. (Hint: derive a formula of  $\frac{d^n}{dx^n}(fg)$  from the product rule.)