Calculus — Homework 4 (Fall 2023)

1. Use the intermediate value theorem to show that there is a solution of the given equation in the indicated interval.
   (a) $2x^3 - 4x^2 + 5x - 4 = 0, \quad [1, 2]$.
   (b) $\sin x + 2\cos x - x^2 = 0, \quad [0, \pi/2]$.

2. (Brouwer fixed-point theorem.) Show that if f is continuous on $[0, 1]$ and $0 \leq f(x) \leq 1$ for all x in $[0, 1]$, then there exists at least one point c in $[0, 1]$ at which $f(c) = c$. (HINT: Apply the intermediate value theorem to the function $g(x) = x - f(x)$.)

3. True or false? Explain how your answers are consistent with the extreme value theorem
   (a) The function $f(x) = x^2$ attains a maximum value on $[-1, 1]$.
   (b) The function $f(x) = x^2$ attains a minimum value on $[-1, 1]$.
   (c) The function $f(x) = x^2$ attains a maximum value on $(-1, 1)$.
   (d) The function $f(x) = x^2$ attains a minimum value on $(-1, 1)$.
   (e) The function $f(x) = x^2$ is bounded on $(-1, 1)$.

4. Draw the graph of f; indicate where f is not differentiable, and indicate where f is not continuous.
   (a) $f(x) = \sqrt{|x|}$.
   (b) $f(x) = |x^2 - 4|$.
   (c) $f(x) = \begin{cases} x^2, & |x| \leq 1, \\ 2 - x, & |x| > 1. \end{cases}$