Calculus — Homework 2 (Fall 2023)

1. True or false? Explain your answers.
   (a) If $\lim_{x\to c}\Big(f(x)+g(x)\Big)$ exists but $\lim_{x\to c} f(x)$ does not exist, then $\lim_{x\to c} g(x)$ does not exist.
   (b) If $\lim_{x\to c}\Big(f(x)+g(x)\Big)$ and $\lim_{x\to c} f(x)$ exist, then $\lim_{x\to c} g(x)$ exists.
   (c) If $\lim_{x\to c}\sqrt{f(x)}$ exists, then $\lim_{x\to c} f(x)$ exists.
   (d) If $\lim_{x\to c} f(x)$ exists, then $\lim_{x\to c}\sqrt{f(x)}$ exists.
   (e) If $\lim_{x\to c} f(x)$ exists, then $\lim_{x\to c}\dfrac{1}{f(x)}$ exists.
   (f) If $\lim_{x\to c}\dfrac{f(x)}{g(x)}$ exists and $\lim_{x\to c} g(x)=0$, then $\lim_{x\to c} f(x)=0$.

2. Evaluate the limit $\lim_{x\to 0}|x|\cos(\dfrac{1}{x^2})$. Prove your answer.

3. Evaluate the limits that do exist. If the limit does not exist, explain why.
   (a) $\lim_{x\to 0}\dfrac{\sin 3x}{x}$.
   (b) $\lim_{x\to 0}\dfrac{3x}{\sin 5x}$.
   (c) $\lim_{x\to 0}\dfrac{\sin 4x}{\sin 5x}$.
   (d) $\lim_{x\to 0}\dfrac{\sin(x^2)}{x}$.
   (e) $\lim_{x\to 0}\dfrac{\sin^2(x^2)}{x^2}$.
   (f) $\lim_{x\to 0}\dfrac{\tan^2(3x)}{4x^2}$.
   (g) $\lim_{x\to 0}\dfrac{\cos x-1}{2x}$.
   (h) $\lim_{x\to \pi/4}\dfrac{\sin x}{x}$.
   (i) $\lim_{x\to \pi}\dfrac{\sin x}{x-\pi}$.

4. Evaluate the limits.
   (a) $\lim_{x\to\infty}\sqrt{\dfrac{8x^2-3}{2x^2+x}}$.
   (b) $\lim_{x\to-\infty}\Big(\dfrac{1-x^3}{x^2+7x}\Big)^5$.
   (c) $\lim_{x\to-\infty}\dfrac{\sqrt{x^2+1}}{x+1}$.
   (d) $\lim_{x\to\infty}\dfrac{2x^{5/3}-x^{1/3}+7}{x^{9/5}+3x+\sqrt{x}}$.

5. Determine whether or not the function is continuous at the indicated point. Explain your answers.
   (a) $f(x)=x^3-5x+1\,,\quad x=2.$
   (b) $f(x)=x\sin x+\cos^2 x\,,\quad x=1.$
   (c) $f(x)=\tan x\,,\quad x=\pi/2.$
   (d) $f(x)=\sqrt{(x-1)^2+5}\,,\quad x=1.$
   (e) $f(x)=|4-x^2|\,,\quad x=2.$
   (f) $f(x)=\begin{cases}x^2+4, & x<2,\\ x^3, & x\ge 2,\end{cases}\quad x=2.$
   (g) $f(x)=\begin{cases}x^2+5, & x<2,\\ x^3, & x\ge 2,\end{cases}\quad x=2.$
   (h) $f(x)=\begin{cases}\dfrac{|x-1|}{x-1}, & x\neq 1,\\ 0, & x=1,\end{cases}\quad x=1.$
   (i) $f(x)=\begin{cases}-x^2, & x<0,\\ 0, & x=0,\\ 1/x^2, & x>0,\end{cases}\quad x=0.$