# Calculus — Homework 12 (Fall 2023)

1. Decompose the rational functions into the form

$$p(x)+\frac{q_1(x)}{(x-b_1)^{r_1}}+\cdots+\frac{q_i(x)}{(x-b_i)^{r_i}}+\frac{q_{i+1}(x)}{(x^2+c_{i+1}x+d_{i+1})^{r_{i+1}}}+\cdots+\frac{q_j(x)}{(x^2+c_jx+d_j)^{r_j}},$$

where $p(x), q_k(x)$ are polynomials, and the degree of $q_k(x)$ is smaller than the degree of denominator.

(a) $\dfrac{1}{x^2+7x+6}$.

(b) $\dfrac{x}{x^4-1}$.

(c) $\dfrac{x^5+x^2}{x^4-1}$.

(d) $\dfrac{x^4}{x^3+x^2-2x}$.

2. Calculate.

(a) $\displaystyle\int \frac{1}{x^2+7x+6}\,dx$.

(b) $\displaystyle\int \frac{x^5}{(x-2)^2}\,dx$.

(c) $\displaystyle\int \frac{1}{x^2-6x+13}\,dx$.

(d) $\displaystyle\int \frac{x}{x^2-6x+13}\,dx$.

(e) $\displaystyle\int \frac{1}{(x^2-2x+2)^2}\,dx$.

(f) $\displaystyle\int \frac{x}{(x^2-2x+2)^2}\,dx$.

(g) $\displaystyle\int \frac{x}{x^4-1}\,dx$.

(h) $\displaystyle\int \frac{x^5+x^2}{x^4-1}\,dx$.

(i) $\displaystyle\int \frac{x^4}{x^3+x^2-2x}\,dx$.

(j) $\displaystyle\int \frac{1}{(x-1)(x^2+1)^2}\,dx$.

3. Determine whether the functions satisfy the differential equation.

(a) $y'+xy=x$; $\quad y_1(x)=e^{-x^2/2}$, $\quad y_2(x)=1+e^{-x^2/2}$.

(b) $y''+4y=0$; $\quad y_1(x)=2\sin 2x$, $\quad y_2(x)=2\cos x$, $\quad y_3(x)=\sinh 2x$.