Calculus — Homework 11 (Fall 2023)

1. Verify the identity
$$\cosh(x+y) = \cosh x\,\cosh y + \sinh x\,\sinh y\,.$$

2. Evaluate.

   (a) $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx.$

   (b) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{4-x^2}}\,dx.$

   (c) $\displaystyle\int_{0}^{3/2} \frac{dx}{9+4x^2}.$

   (d) $\displaystyle\int_{\ln 2}^{\ln 3} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx.$

   (e) $\displaystyle\int_{0}^{\ln 2} \sinh 2x\,dx.$

   (f) $\displaystyle\int_{0}^{\ln 2} x\sinh x\,dx.$

   (g) $\displaystyle\int_{0}^{1} \frac{dx}{5^x}.$

   (h) $\displaystyle\int_{0}^{1} \frac{x^3}{1+x^4}\,dx.$

   (i) $\displaystyle\int_{1}^{2} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx.$

   (j) $\displaystyle\int_{-3}^{-2} \frac{dx}{x^2+6x+10}.$

3. Calculate.

   (a) $\displaystyle\int_{0}^{1} xe^{-x}\,dx.$

   (b) $\displaystyle\int_{0}^{1} x^3e^{-x^2}\,dx.$

   (c) $\displaystyle\int_{1}^{e^2} x\,\ln(\sqrt{x})\,dx.$

   (d) $\displaystyle\int_{0}^{1/2} x\cos\pi x\,dx.$

   (e) $\displaystyle\int_{0}^{1/4} \arcsin 2x\,dx.$

   (f) $\displaystyle\int x^2(e^x-1)\,dx.$

   (g) $\displaystyle\int (\ln x)^2\,dx.$

   (h) $\displaystyle\int e^x\sin x\,dx.$

4. Show that if f and g have continuous second derivatives and $f(a)=g(a)=f(b)=g(b)=0$, then
$$\int_a^b f(x)g''(x)\,dx = \int_a^b g(x)f''(x)\,dx.$$

5. Calculate.

   (a) $\displaystyle\int_{0}^{\pi/6} \sin^2 3x\,dx.$

   (b) $\displaystyle\int_{0}^{\pi} \sin^3 x\,dx.$

   (c) $\displaystyle\int \sin^3 x\cos^3 x\,dx.$

   (d) $\displaystyle\int \sin^2 x\cos^4 x\,dx.$

6. Use integration by parts to show that for $n>2$,
$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx.$$
Then compute
$$\int_0^{\pi/2} \sin^{2k}x\,dx \qquad \text{and} \qquad \int_0^{\pi/2} \sin^{2k+1}x\,dx$$
for $k \geq 1$.

7. Calculate.

(a) $\displaystyle\int_0^2 \frac{x^2}{\sqrt{16-x^2}}\,dx.$

(b) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx.$

(c) $\displaystyle\int \frac{x^2}{\sqrt{x^2-4}}\,dx.$

(d) $\displaystyle\int \frac{x^2}{\sqrt{4+x^2}}\,dx.$

(e) $\displaystyle\int \frac{x}{\sqrt{x^2-2x+3}}\,dx.$

(f) $\displaystyle\int \frac{1}{(x^2+1)^3}\,dx.$