# Calculus — Homework 10 (Fall 2023)

1. Calculate.

   (a) $\dfrac{d}{dx}\left(\sqrt{\log_3 x}\right)$.

   (b) $\displaystyle\int_1^5 \frac{\log_5 x}{x}\,dx$.

2. Find the number(s) $x$ which satisfy the equation.

   (a) $10 = e^x$.

   (b) $\log_x 2 = \log_3 x$.

3. Prove that, if $n$ is a positive integer, then there exists $N$ such that

   $$e^x > x^n$$

   for all $x \geq N$. (This is a famous property. You can find a proof online.)

4. Let

   $$f(x) = \begin{cases} e^{-1/x^2}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

   Is $f$ differentiable at $x = 0$? Is $f$ twice differentiable at $x = 0$? Justify your answers.

5. Evaluate.

   (a) $\displaystyle\lim_{x\to\infty} \left(\frac{x+1}{x-1}\right)^x$.

   (b) $\displaystyle\lim_{x\to 0^+} x^x$.

6. Determine the exact value.

   (a) $\arcsin(-\sqrt{3}/2)$.

   (b) $\cos(\arctan 2)$.

   (c) $\arcsin\big(\sin(11\pi/6)\big)$.

   (d) $\cos\big(2\arcsin(4/5)\big)$.

7. Differentiate.

   (a) $f(x) = \arctan(x+1)$.

   (b) $f(x) = e^x \arcsin x$.

8. Let $a > 0$. Calculate

   $$\int \frac{1}{a^2 + (x+b)^2}\,dx.$$