Complex Analysis — Homework 9

1. Evaluate the following integrals

   (a) $\displaystyle\int_0^\infty \frac{x^2}{(x^2+4)^2(x^2+9)}\,dx,$

   (b) $\displaystyle\int_0^\infty \frac{\sin x}{x(1+x^2)}\,dx,$

   (c) $\displaystyle\int_0^\infty \frac{\cos x}{1+x^2}\,dx,$

   (d) $\displaystyle\int_0^\infty \frac{1}{x^3+8}\,dx,$

   (e) $\displaystyle\int_0^\infty \frac{1}{\sqrt[3]{x}(1+x)}\,dx,$

   (f) $\displaystyle\int_0^{2\pi} \frac{\sin^2 x}{5+3\cos x}\,dx,$

   (g) $\displaystyle\int_0^{\pi} \frac{dx}{a+\cos x},\ a\in\mathbb{R},\ |a|>1.$

2. Evaluate
$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx.$$
   (Hint: Integrate $(e^{2iz}-1-2iz)/z^2$ around a large semi-circle.)

3. Evaluate
$$\int_0^\infty \frac{1}{1+x^n}\,dx,$$
   where $n \geq 2$ is a positive integer.

4. Suppose $f = P/Q$, where P and Q are polynomials with $\deg Q - \deg P \geq 2$. Show that the sum of the residues of f is zero.

5. Let $\mathbb{R}_{\leq 0} = \{x \in \mathbb{R} : x \leq 0\} \subset \mathbb{C}$ and $U = \mathbb{C} - \mathbb{R}_{\leq 0}$. Evaluate
$$\int_\gamma e^z \log z\,dz,$$
   where $\log z$ is the branch in U for which $\log 1 = 0$ and γ is the parabola: $\gamma(t) = 1 - t^2 + it$, $-\infty < t < \infty$.