Complex Analysis — Homework 11

1. Find a conformal mapping f from S onto T, where
   (a) $S = \{z = x + iy : -2 < x < 1\}$; $T = D(0;1)$;
   (b) $S = T =$ the upper half plane; $f(-2) = -1$, $f(0) = 0$ and $f(2) = 2$;
   (c) $S = \{re^{i\theta} : r > 0,\ 0 < \theta < \pi/4\}$; $T = \{x + iy : 0 < y < 1\}$;
   (d) $S = D(0;1) - [0,1]$; $T = D(0;1)$.
   (Notation: $[0,1] = \{z = x + iy : 0 \leq x \leq 1,\ y = 0\} \subset \mathbb{C}$.)

2. What is the image of the upper half plane under a mapping of the form

$$f(z) = \frac{az+b}{cz+d}, \quad a,\ b,\ c,\ d \text{ real}; \qquad ad - bc < 0?$$

3. Find a formula for all the automorphisms of the first quadrant.

4. Given a conformal mapping f of R onto the unit disc U and $z_0 \in R$, find a conformal mapping g of R onto U with $g(z_0) = 0$ and $g'(z_0) > 0$.

5. Let R be a simply connected domain and assume $z_1, z_2 \in R$. Show that there exists a conformal mapping of R onto itself, taking z_1 to z_2. (Consider two cases: $R \neq \mathbb{C}$ and $R = \mathbb{C}$.)

6. Let $R \subset \mathbb{C}$ be the open set

$$R = \{z \in \mathbb{C} : |z-1| < 1 \text{ and } |z-i| < 1\}.$$

   Find a conformal mapping from R onto the unit disk U.