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Approximation Using Generalization Of Lupaş Operators*

Shruti Shailesh Kariya†, Rajiv Biharilal Gandhi‡, Harsh Paresh Kothari¶, Prashantkumar Patel‖, Vishnu Narayan Mishra**

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Abstract

In this paper, a generalization of Lupaş operators is defined which establishes better convergence. This generalized Lupaş operator satisfies the Korovkin conditions for density theorems for the extended value of parameter a, and also leads to better approximation results. This generalization removes the limitation of Lupaş operators which satisfies Korovkin conditions for only one value of a, *i.e.*, $a = 1/2$. The rate of convergence and approximation properties are discussed with proofs. Graphical representations are also included to show better performance of generalized operators with the original operators.

1 Introduction

Lupaş [1] presented the following operators

$$L_m(\rho\,;y) = (1-a)^{my} \sum_{j=0}^{\infty} \frac{(my)_j}{j!}\, a^j \,\rho\left(\frac{j}{m}\right),\ |a|<1,\ y\geq 0, \tag{1}$$

for $\rho : \mathbb{R}^+ \to \mathbb{R}, m \in \mathbb{N}$ and $(my)_0 = 1$ and $(my)_j = (my)(my+1)\cdots(my+j-1),\ j \in \mathbb{N}$, where for $my = \beta$,

$$\frac{1}{(1-a)^\beta} = \sum_{j=0}^{\infty} \frac{(\beta)_j}{j!}\, a^j,\ |a|<1.$$

Agratini [2] studied operators (1) and found that for these operators to satisfy $\lim_{m\to\infty} L_m(e_i; y) = e_i(y)$, where $e_i(y) = y^i,\ i = 0, 1, 2$, possible value for a is $a = 1/2$. Agratini [2] defined the following operators, by choosing $a = 1/2$ in (1),

$$L_m(\rho\ ; y) = 2^{-my} \sum_{j=0}^{\infty} \frac{(my)_j}{2^j j!} \rho\left(\frac{j}{m}\right),\ m \in \mathbb{N},\ y \geq 0. \tag{2}$$

As shown by Agratini [2], by fixing $p > 0$ and considering H_p as mappping from $C([0,\infty))$ into $C([0,p])$, defined by $H_p(g) = g|_{[0,p]}$, it is the case that for L_m operators defined by (1), $H_p(L_m e_i)$ converges uniformly to $H_p(e_i)$ on $[0,p]$, where $i = 0, 1, 2$. Also, using the test functions, $e_i(y) = y^i$, for $i = 0, 1, 2$, and Korovkin-type results [[3],[4]], Agratini [2] proved the density theorem for convergence of operators given by (2) as mentioned below:

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†Research Scholar, Gujarat Technological University, Ahmedabad, Gujarat, India
‡Department of Mathematics, BVM Engineering College, Vallabh Vidyanagar, Gujarat, India. Corresponding author.
§Research Scholar, Gujarat Technological University, Ahmedabad, Gujarat, India
¶Department of Mathematics, A. D. Patel Institute of Technology, The Charutar Vidya Mandal (CVM) University, Vallabh Vidyanagar, Gujarat, India
‖Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar, Gujarat, India
**Department of Mathematics, Faculty of Science, Indira Gandhi National Tribal University, Lalpur, Amarkantak, Madhya Pradesh, India

Let L_m be as defined by (2), then

$$\lim_{m\to\infty} L_m(\rho\,; y) = \rho(y)$$

uniformly on $[0, p]$ for any $p > 0$.

The aim of all such work carried out is to get as much better approximation as possible, to the Weistrass-type of approximation process. Many modifications were proposed with the same objective. For a better idea of such modifications, one can refer to [5–8, 10–31]. Lupaş operators proposed by Agratini [2] are conditioned to have only one value of the parameter a, i.e., $a = 1/2$ to satisfy the Korovkin-type theorem for uniform convergence of the operators. The generalization of Lupaş discussed in this study removes that constraint.

The present work is structured into sections as follows. The definition of the generalization of Lupaş operators and its value at test functions and moments are discussed in section 2. In section 3, the Korovkin-type theorem is discussed and proved for this operator. The rate of convergence is discussed in section 4. Also, we prove density theorems using the modulus of smoothness. In section 5, asymptotic formulas for this generalization of Lupaş operators are given. In section 6, we represent Lupaş operator and its generalization at different values of m by graphical method. In the last section, we present some future scopes.

2 Construction of Operators

We establish a generalization of Lupaş operators as follows:

$$L^*_{m,a}(\rho\,; y) = (1-a)^{mr(y)} \sum_{j=0}^{\infty} \frac{(mr(y))_j}{j!}\, a^j\, \rho\left(\frac{j}{m}\right),\ |a| < 1,\ a \neq 0,\ y \geq 0, \tag{3}$$

where $\rho : \mathbb{R}^+ \to \mathbb{R}$, $m \in \mathbb{N}$ and $r(y)$ is a function defined as,

$$r(y) = \frac{(1-a)y}{a},\ a \text{ is a parameter.}$$

It is to be observed that by putting $a = 1/2$ in this operator, we get the original Lupaş operator mentioned in (2) which was proposed by Agratini [2]. This paper is focused on the detailed study of these operators defined by (3). The following Lemmas 1 and 2 are mentioned with proof and they are necessary to support our key results.

Lemma 1 *For the operators defined by* (3) *and for test functions* $e_i(y) = y^i$, *we have*

1. $L^*_{m,a}(e_0\,; y) = 1,$
2. $L^*_{m,a}(e_1\,; y) = y,$
3. $L^*_{m,a}(e_2\,; y) = y^2 + \dfrac{y}{(1-a)m},$
4. $L^*_{m,a}(e_3\,; y) = \dfrac{y(a+1)}{(1-a)^2m^2} + \dfrac{3y^2}{(1-a)m} + y^3.$

Proof.

1. *For* $e_0(y) = y^0 = 1$, *we have*

$$L^*_{m,a}(e_0\,; y) = (1-a)^{mr(y)} \sum_{j=0}^{\infty} \frac{(mr(y))_j}{j!} a^j = (1-a)^{mr(y)}(1-a)^{-mr(y)} = 1.$$

2. *For $e_1(y) = y$, we get*

$$\begin{aligned}
L^*_{m,a}(e_1;y) &= (1-a)^{mr(y)} \sum_{j=0}^{\infty} \frac{(mr(y))_j}{j!} a^j \left(\frac{j}{m}\right) \\
&= \frac{(1-a)^{mr(y)}}{m} \sum_{j=1}^{\infty} \frac{(mr(y))_j}{(j-1)!} a^j \\
&= \frac{a(1-a)^{mr(y)}}{m} \sum_{j=1}^{\infty} \frac{(mr(y))(mr(y)+1)_{j-1}}{(j-1)!} a^{j-1},
\end{aligned}$$

replacing j by $j+1$, we get

$$\begin{aligned}
L^*_{m,a}(e_1;y) &= \frac{a(1-a)^{mr(y)}}{m} \sum_{j=0}^{\infty} \frac{((mr(y))(mr(y)+1)_j}{j!} a^j \\
&= \frac{a(1-a)^{mr(y)}(mr(y))}{m} (1-a)^{-(mr(y)+1)} \\
&= a\left(\frac{(1-a)y}{a}\right)(1-a)^{-1} \\
&= y.
\end{aligned}$$

3. *Considering $e_2(y) = y^2$,*

$$\begin{aligned}
L^*_{m,a}(e_2;y) &= (1-a)^{mr(y)} \sum_{j=0}^{\infty} \frac{(mr(y))_j}{j!} a^j \left(\frac{j^2}{m^2}\right) \\
&= \frac{a(1-a)^{mr(y)}}{m^2} \sum_{j=1}^{\infty} \frac{(mr(y))(mr(y)+1)_{j-1}}{(j-1)!} a^{j-1}((j-1)+1) \\
&= \frac{a(1-a)^{mr(y)}}{m^2} \sum_{j=0}^{\infty} \frac{(mr(y))(mr(y)+1)_j}{j!} a^j (j) + \frac{a(1-a)^{mr(y)} mr(y)}{m^2} \sum_{j=0}^{\infty} \frac{(mr(y)+1)_j}{(j)!} a^j \\
&= \frac{a^2(1-a)^{mr(y)} mr(y)}{m^2} \sum_{j=1}^{\infty} \frac{(mr(y)+1)_j}{(j-1)!} a^{j-1} + \frac{a(1-a)^{mr(y)} mr(y)}{m^2} (1-a)^{-(mr(y)+1)} \\
&= \frac{a^2(1-a)^{mr(y)} mr(y)}{m^2} \sum_{j=1}^{\infty} \frac{(mr(y)+1)(mr(y)+2)_{j-1}}{(j-1)!} a^{j-1} + \frac{y}{m} \\
&= \frac{a^2(1-a)^{mr(y)} mr(y)(mr(y)+1)}{m^2} \sum_{j=0}^{\infty} \frac{(mr(y)+2)_j}{(j)!} a^j + \frac{y}{m} \\
&= \frac{a^2(1-a)^{mr(y)} mr(y)(mr(y)+1)}{m^2} (1-a)^{-(mr(y)+2)} + \frac{y}{m} \\
&= y^2 + \frac{ay}{(1-a)m} + \frac{y}{m} \\
&= y^2 + \frac{y}{(1-a)m}.
\end{aligned}$$

4. *Considering $e_3(y) = y^3$, the result follows from the calculations similar to the previous calculations.*

■

Lemma 2 *For the operators defined by (3) and $\mu_i^y(t) = (t-y)^i$, moment estimates are given as follows,*

1. $L^*_{m,a}(\mu_0^y(t);y) = 1$,
2. $L^*_{m,a}(\mu_1^y(t);y) = 0$,
3. $L^*_{m,a}(\mu_2^y(t);y) = \dfrac{y}{(1-a)m}$,
4. $L^*_{m,a}(\mu_3^y(t);y) = \dfrac{y(a+1)}{(1-a)^2m^2}$.

Proof. *The claims can be proved by using linearity property of opearators* (3) *and results of Lemma 1.* ■

The convergence of operators is discussed in next section.

3 Korovkin-type Theorem

The Korovkin and Korovkin-type theorems [3, 4] are results to establish uniform convergence for a sequence of linear positive operators on a function space given that the sequence converges uniformly for the functions $e_i(y) = y^i$, $i = 0, 1, 2$, called test functions.

Theorem 1 *Let $\rho \in C([0,p])$ and $L^*_{m,a}(\rho\,; y)$ be as defined in* (3). *Then*

$$\lim_{m\to\infty} L^*_{m,a}(\rho\,;y) = \rho(y)$$

uniformly on $[0,p]$ for any $p > 0$.

Proof. As proved in Lemma 1,

$$L^*_{m,a}(e_o;y) = e_0(y), \quad L^*_{m,a}(e_1;y) = e_1(y), \quad \lim_{m\to\infty} L^*_{m,a}(e_2;y) \to e_2(y)$$

uniformly on $[0,p]$. Therefore, by the Korovkin-type theorem, $L^*_{m,a}(\rho\,;y)$ converges to ρ uniformly for all $\rho \in C([0,p])$. ■

4 Rate of Convergence

To establish results on how fast the sequence of linear positive operators converges to the function, it operates upon, moduli of smoothness is a tool. The moduli of smoothness of order one and two are defined as,

$$w_b(\rho,r) = \sup\{\left|\Delta_a^b\rho(y)\right| : |a| \le r,\ y, y+ab \in A\}, \quad r > 0,$$

where $b \in \{1,2\}$, $\rho : A \to \mathbb{R}$ ($A \subseteq \mathbb{R}$) is a bounded real function, $\Delta_a^1\rho(y) = \rho(y+a) - \rho(y)$ and $\Delta_a^2\rho(y) = \Delta_a^1\rho(y+a) - \Delta_a^1\rho(y)$.

Theorem 2 *For $L^*_{m,a}$ defined by* (3) *on $C([0,p])$ and $p > 0$, the following inequality holds true:*

$$\left|(L^*_{m,a}\rho)(y) - \rho(y)\right| \le w_1\left(\rho\ , \frac{1}{\sqrt{m}}\right)\left(1 + \sqrt{\frac{p}{1-a}}\right).$$

Proof. We have

$$|\rho(t) - \rho(y)| \le w_1(\rho, |t-y|) \le \left(1 + \frac{|t-y|}{r}\right) w_1(\rho, r), \tag{4}$$

for all $y, t \in [0,p]$ and $r > 0$. By using this inequality and previously mentioned definitions,

$$\left|L^*_{m,a}(\rho\,;y) - \rho(y)\right| \le L^*_{m,a}(1\,;y) w_1(\rho,r) + \frac{1}{r} L^*_{m,a}(|t-y|\ ;y) w_1(\rho,r). \tag{5}$$

Now, using the Cauchy-Schwarz inequality

$$L^*_{m,a}(|t-y|\,;y) \le \alpha_m(y)(L^*_{m,a}(1\,;y))^{1/2}, \tag{6}$$

where $\alpha_m(y)^2 = L^*_{m,a}((t-y)^2;y)$. Using (5) and (6), we get

$$|L^*_{m,a}(\rho\,;y) - \rho(y)| \le w_1(\rho,r)\left(1 + \frac{1}{r}\sqrt{\frac{p}{(1-a)m}}\right).$$

By choosing $r = 1/\sqrt{m}$, we get the desired inequality. ■

Remark 1 *By taking $a = 1/2$ in above result, we get the the result proved in [2] for Lupaş operators. By choosing value of a, where $|a| < \frac{1}{2}$, we get better results than that of Lupaş operators (1) in a sense that error reduces.*

Theorem 3 *For $L^*_{m,a}$ defined as (3) on $C([0,p])$ where $p > 0$ and ρ is continuously differentiable on $[0,p]$, the following inequality is derived*

$$|(L^*_{m,a}\rho)(y) - \rho(y)| \le \left(\sqrt{\frac{p}{(1-a)m}} + \frac{p}{(1-a)\sqrt{m}}\right) w_1\left(\rho', \frac{1}{\sqrt{m}}\right).$$

Proof. As ρ is continuously differentiable on $[0,p]$, by the mean-value theorem,

$$\rho(a) - \rho(b) = (a-b)\rho'(y) + (a-b)(\rho'(t) - \rho'(y)),$$

for every $a, b \in [0,p]$ and $a < t < b$. Using 4 for ρ' and Cauchy-Schwarz inequality for further simplification, we get

$$\begin{aligned}
\left|(L^*_{m,a}\rho)(y) - \rho(y)\right| &\le \left(L^*_{m,a}(|t-y|\,;y) + \frac{1}{r}L^*_{m,a}(|t-y|^2\,;y)\right) w_1(\rho',r)\\
&\le \left(L^*_{m,a}(|t-y|^2\,;y)\right)^{1/2}\left(1 + \frac{1}{r}(L^*_{m,a}(|t-y|^2\,;y))^{1/2}\right) w_1(\rho',r)\\
&\le \sqrt{\frac{p}{m(1-a)}}\left(1 + \frac{1}{r}\sqrt{\frac{p}{m(1-a)}}\right) w_1(\rho',r).
\end{aligned}$$

After choosing $r = 1/\sqrt{m}$, we get the desired inequality. ■

Next, we obtain another estimate that involves a second-order modulus of smoothness.

Theorem 4 *For $L^*_{m,a}$ defined as (3) on $C([0,p])$ and $p > 0$, the following inequality holds:*

$$|L^*_{m,a}(\rho\,;y) - \rho(y)| \le \left(3 + \frac{p}{(1-a)}\max\left\{1, \frac{p}{m}\right\}\right) w_2\left(\rho, \frac{1}{\sqrt{m}}\right).$$

Proof. This estimate is based upon a theorem by Gonska [9, page 331, theorem 4.1]. So, it can be proved by considering Lemma 1 and above mentioned result by Gonska. ■

Remark 2 *It is to be noticed that if we take $a = 1/2$ in all the above identities proved, we get the results proved for Lupaş operators mentioned in [2].*

5 Voronovskaja-type Theorem

The asymptotic formula for generalized operators, viz., Voronovskaja-type theorem is established in this section.

Theorem 5 *Let the operators* $L^*_{m,a}$ *be defined by* (3) *and* $\rho \in C([0,\infty))$. *If* $\rho''(y)$ *exists at some point* $y > 0$ *and* $\rho(t) = O(t^2)$ *as* $t \to 0$, *then*

$$\lim_{m\to\infty} m((L^*_{m,a}\rho)(y) - \rho(y)) = \frac{y}{1-a}\rho''(y).$$

Proof. We use Taylor's expansion to prove the desired identity. By Taylor's theorem

$$\rho(t) - \rho(y) = (t-y)\rho'^2 \left(\frac{1}{2}\rho''(y) + \epsilon_y(t)\right),$$

where ϵ is bounded and $\lim\limits_{t\to 0} \epsilon_y(t) = 0$. After applying $L^*_{m,a}$ operator on above equation, we get

$$(L^*_{m,a}\rho)(y) - \rho(y) = L^*_{m,a}(t-y\,;y)\rho'(y) \;+\; \frac{1}{2}L^*_{m,a}((t-y)^2\,;y)\rho''(y) + (L^*_{m,a}\alpha_y)(y), \tag{7}$$

where $\alpha_y = (t-y)^2\epsilon_y(t)$. Using Cauchy-Schwarz inequality, we get the following.

$$(L^*_{m,a}\alpha_y)(y) \le \left(L^*_{m,a}((t-y)^4; y)\right)^{1/2} \left(L^*_{m,a}(\epsilon^2_y(t); y)\right)^{1/2}.$$

which leads to $\lim\limits_{m\to 0} m(L^*_{m,a}\alpha_y)(y) = 0$. Lemma 2 using in (7) gives,

$$\lim_{m\to\infty} m((L^*_{m,a}\rho)(y) - \rho(y)) = \frac{y}{1-a}\rho''(y).$$

■

6 Graphical Representation

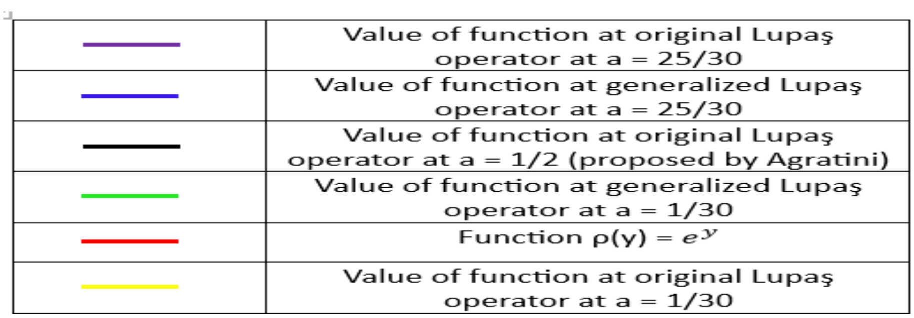

Figure 1: Identification of all the curves in graphs for $m = 10, 30$ and 50

As mentioned earlier, this generalization is proposed with the intention to allow values of the parameter of a other than $1/2$. We have represented the comparison between original Lupaş operators and generalized operators for different values of a and m through graphical representation. The following table is to identify the curves for all the graphs of Lupaş operator and its modification for different values of m, i.e., $m = 10$, 30 and 50. We can observe from these graphs that as the value of m increases, all the curves of operators tend to get closer to the curve of the function. As mentioned earlier, Lupaş operators give the best convergence for value $a = 1/2$ proposed by Agratini [2]. It is verified here from this representation that the farthest curves from the graph of function are graphs of Lupaş operators at other values of a than $1/2$. It is observed that for $0 < a < 1/2$, graphs of generalization of operators (3) are closer than graphs at $1/2 < a < 1$. For $a = 1/2$ in this generalization, we get the original Lupaş operators mentioned in (2). The graphs show that this generalization provides a better approximation than the original Lupaş operators.

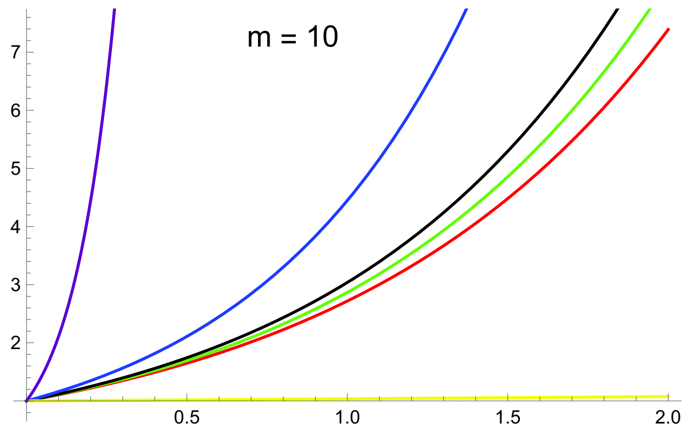

Figure 2: Graphs of (1) for $a = 25/30$ (Purple), of (3) for $a = 25/30$ (Blue), of (1) for $a = 1/2$ (Black), of (3) for $a = 1/30$ (Green), of function $\rho(y) = e^y$ (Red) and of (1) for $a = 1/30$ (Yellow) for $m = 10$.

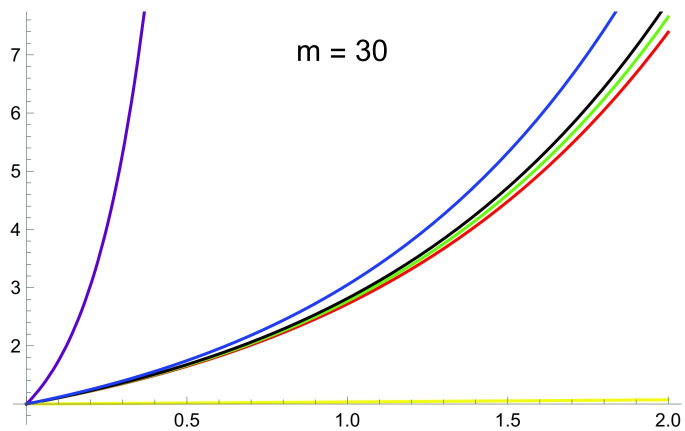

Figure 3: Graphs of (1) for $a = 25/30$ (Purple), of (3) for $a = 25/30$ (Blue), of (1) for $a = 1/2$ (Black), of (3) for $a = 1/30$ (Green), of function $\rho(y) = e^y$ (Red) and of (1) for $a = 1/30$ (Yellow) for $m = 30$.

7 Future Scope

We have proposed the generalization of Lupaş operators and discussed some of the approximation properties of it. More properties can be derived for the same operators for further work. Also, modification of this operator can be derived which leads to even better approximation by means of the rate of convergence.

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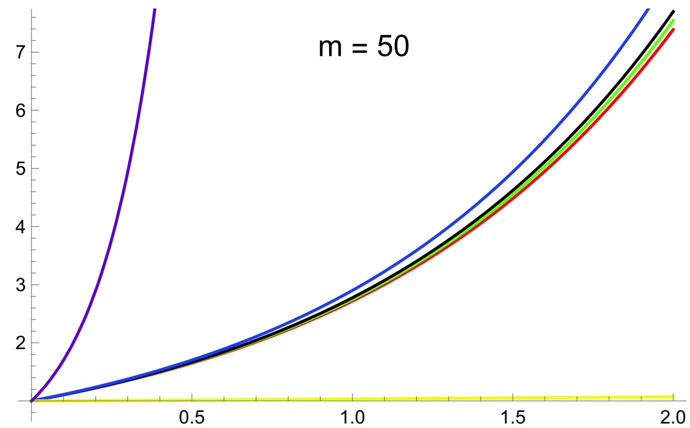

Figure 4: Graphs of (1) for $a = 25/30$ (Purple), of (3) for $a = 25/30$ (Blue), of (1) for $a = 1/2$ (Black), of (3) for $a = 1/30$ (Green), of function $\rho(y) = e^y$ (Red) and of (1) for $a = 1/30$ (Yellow) for $m = 50$.

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