

Generalised Arithmetic Mean-Geometric Mean Inequality And Its Application To Find The Optimal Policy Of The Classical EOQ Model Under Interval Uncertainty*

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Abstract

The present work aims to introduce a new interval order relation and generalized arithmetic mean-geometric mean (AM-GM) inequality for interval numbers. Again, this work tries to implement this generalized AM-GM inequality in the extended Harris-Wilson's EOQ model to derive the optimal policy. The Harris-Wilson's model is extended by taking interval-valued demand rate, holding cost, and ordering cost as interval-valued instead of real-valued. Finally, the optimality condition of the extended EOQ model is validated by some numerical examples.

1 Introduction

Interval order relation plays a vital role in solving the real-life decision-making problems (optimization problems) under interval uncertainty. Thus, interval order relation is intimately involved with the interval optimization problems. Moore [1] first proposed the idea of interval order relation. After that, Ishibuchi and Tanaka [2] introduced a number of definitions of interval order relations. Succeeding them, Hu and Wang [3], Mahato and Bhunia [4], Bhunia and Samanta [5], and others proposed several types of interval order relations. According to Bhunia and Samanta [5], the interval order relation proposed by them is more significant than the others. Although Bhunia and Samanta's [5] interval order relations are widely used in the research works on inventory control problems under interval uncertainty, their proposed definitions have some drawbacks. And these are pointed out in the later section of this manuscript.

On the other hand, arithmetic mean-geometric mean (AM-GM) inequality also works as a derivative free optimizer tool. In the area of inventory control, Grubbstrom [6] first used the AM-GM inequality to derive the optimal policy of the classical Harris-Wilson EOQ model (Harris [7]). After that, Grubbstrom and Erdem [8] again used the AM-GM inequality to find the optimality conditions of extended EOQ model by allowing the shortages. Cardenas-Barron [9] also used the same approach for finding the optimal policy of the classical economic production quantity (EPQ) model. Also, to study the optimal policy of the EOQ model under uncertainty, Gani et al. [10] considered all the uncertain inventory parameters as fuzzy numbers and used the AM-GM inequality to find the optimality conditions. On the other hand, few works on inventory problems with interval uncertainty are available in the literature. Among those, some notable works were reported by Rahman et al. ([11]–[13]), Manna et al. [14], Das et al. [15] and others. However, till now, no one used the AM-GM approach to obtain the optimal policy of the EOQ model under interval uncertainty.

In the present work, a new interval order relation is proposed. And with the help of the proposed interval order relation, the generalized AM-GM inequality for interval numbers is derived. After that, this generalized AM-GM inequality is applied in the classical inventory model under interval uncertainty to obtain the optimal cycle length, optimal economic order quantity, and optimal average cost. Finally, the optimal policy of the said inventory model is illustrated with the help of some numerical examples.

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2 Basic Concepts of Interval

An interval number is defined by the closed and bounded interval as $A = [a_L, a_U] = \{x : a_L \leq x \leq a_U\}$, where a_L and a_U are lower and upper bounds respectively. Any real number can be expressed as an interval number $[x, x]$ as degenerate with zero width. Also, interval number can be expressed in terms of centre and radius form as

$$A = \langle a_c, a_r \rangle = \{x : a_c - a_r \leq x \leq a_c + a_r, x \in \mathbb{R}\},$$

where

$$a_c = \frac{a_U + a_L}{2} \text{ and } a_r = \frac{a_U - a_L}{2}.$$

2.1 Different Arithmetic Operations

Let us consider $A = [a_L, a_U]$ and $B = [b_L, b_U]$ be any two interval numbers. Then arithmetical operations such as addition, subtraction, scalar multiplication, multiplication and division of interval numbers are given below:

(i) **Addition:**

$$A + B = [a_L, a_U] + [b_L, b_U] = [a_L + b_L, a_U + b_U].$$

(ii) **Subtraction:**

$$A - B = [a_L, a_U] - [b_L, b_U] = [a_L - b_U, a_U - b_L].$$

(iii) **Scalar multiplication:**

$$\lambda A = \lambda [a_L, a_U] = \begin{cases} [\lambda a_L, \lambda a_U], & \text{if } \lambda \geq 0, \\ [\lambda a_U, \lambda a_L], & \text{if } \lambda < 0. \end{cases}$$

(iv) **Multiplication:**

$$AB = [\min \{a_L b_L, a_L b_U, a_U b_L, a_U b_U\}, \max \{a_L b_L, a_L b_U, a_U b_L, a_U b_U\}].$$

(v) **Division:**

$$\frac{A}{B} = A \times \left(\frac{1}{B}\right) = [a_L, a_U] \times \left[\frac{1}{b_U}, \frac{1}{b_L}\right], 0 \notin B.$$

(vi) **n-th power:**

$$A^n = [a_L^n, a_U^n], a_L \geq 0, n \in \mathbb{N}.$$

(vii) **n-th root:**

$$A^{1/n} = [a_L^{1/n}, a_U^{1/n}], \text{ for } a_L \geq 0, n \in \mathbb{N}.$$

3 Interval Order Relation

The definitions of interval order relations between two interval numbers have been proposed by several researchers. Here, Bhunia and Samanta's approach [5] of interval order relations along with our proposed approach are discussed.

3.1 Bhunia and Samanta' Approach [5]

Bhunia and Samanta [5] proposed the definitions of interval order relations for maximization problems and minimization problems separately.

Definition 1 Let $A = [a_L, a_U] = \langle a_c, a_r \rangle$ and $B = [b_L, b_U] = \langle b_c, b_r \rangle$ be two intervals. Then, the interval order relation \geq^{\max} between two intervals A and B for maximization problems is as follows:

$$A \geq^{\max} B \Leftrightarrow \begin{cases} a_c \geq b_c, \text{ if } a_c \neq b_c, \\ a_r \leq b_r, \text{ if } a_c = b_c. \end{cases}$$

Definition 2 The interval order relation \leq^{\min} between two intervals A and B for minimization problems is as follows:

$$A \leq^{\min} B \Leftrightarrow \begin{cases} a_c \leq b_c, \text{ if } a_c \neq b_c, \\ a_r \leq b_r, \text{ if } a_c = b_c. \end{cases}$$

Remark 1 In the definition of interval order relation of Bhunia and Samanta [5], it is observed that two different order relations are introduced for maximization problem and minimization problem separately. But order relation should not vary from problem to problem, i.e., it should be problem independent.

3.2 Proposed Approach

In this subsection, we propose the definition of interval order relation that can be used in both maximization and minimization problems.

Definition 3 Let $A = [a_L, a_U] = \langle a_c, a_r \rangle$ & $B = [b_L, b_U] = \langle b_c, b_r \rangle$ be two intervals. Then A is greater than or equal to B if

$$A \geq_C^U B \Leftrightarrow \begin{cases} a_c \geq b_c, \text{ if } a_c \neq b_c, \\ a_U \geq b_U, \text{ if } a_c = b_c. \end{cases}$$

Definition 4 Let A and B be two intervals. Then $A \leq_C^U B$ iff $B \geq_C^U A$.

3.3 Comparison of Proposed Definition with Bhunia and Samanta's Definition

To illustrate the proposed definition of interval order relation and to compare with the Bhunia and Samanta's definition [5], three pairs of numerical examples for A and B are considered by assuming disjoint, partial over lapping and fully over lapping cases. The illustrations and comparisons are presented in the Table 1. From the case III, it is observed that A is selected for both the maximization and minimization case according to the Bhunia and Samanta's definition. In reality, it does never happen for any complete order relation. So, it is a drawback of this definition. However, our proposed definition recovers this drawback.

4 Generalised Arithmetic Mean (AM) and Geometric Mean (GM) Inequality for Intervals

In this section, arithmetic and geometric mean inequality for interval numbers has been derived. The definitions of arithmetic and geometric mean for interval numbers are similar to the usual definitions.

Theorem 1 Let $\{A_i = [a_{iL}, a_{iU}] : a_{iL} \geq 0, i = 1, 2, \dots, n\}$ be the set of n non-negative interval numbers. Then, $\frac{A_1 + A_2 + \dots + A_n}{n} \geq_C^U \sqrt[n]{A_1 A_2 \dots A_n}$ and the equality holds iff $A_1 = A_2 = \dots = A_n$.

Case	Considered Ex-amples	Bhunia and Samanta's de-finition	Proposed definition	Remarks
I	$A = [4, 8]$ and $B = [10, 12]$	Since $a_c = 6 < 11 = b_c$, $A \leq^{\min} B, B \geq^{\max} A$	Since $a_c = 6 < 11 = b_c$, $A \leq_C^U B, B \geq_C^U A$	Each definition gives the same results
II	$A = [4, 8]$ and $B = [7, 11]$	Since $a_c = 6 < 9 = b_c$, $A \leq^{\min} B, B \geq^{\max} A$	Since $a_c = 6 < 9 = b_c$, $A \leq_C^U B, B \geq_C^U A$	Each definition gives the same results
III	$A = [4, 8]$ and $B = [5, 7]$	Since $a_c = 6 = b_c$ and $a_r = 2 > 1 = b_r$, $A \leq^{\min} B, A \geq^{\max} B$	Since $a_c = 6 = b_c$ and $a_U = 8 < 12 = b_c$, $A \leq_C^U B, B \geq_C^U A$	Bhunia and Samanta's definition selects the same interval for both maximization and minimization case. But our proposed definition selects A for minimization case and B for maximization case.

Table 1: Comparison of proposed definition with Bhunia and Samanta's definition

Proof. Using the addition and scalar multiplication of interval numbers, $\frac{A_1+A_2+\dots+A_n}{n}$ can be written as,

$$\frac{A_1 + A_2 + \dots + A_n}{n} = \left[\frac{a_{1L} + a_{2L} + \dots + a_{nL}}{n}, \frac{a_{1U} + a_{2U} + \dots + a_{nU}}{n} \right].$$

Again, using the multiplication and n-th root of interval numbers, $\sqrt[n]{A_1 A_2 \dots A_n}$ can be written as follows:

$$\sqrt[n]{A_1 A_2 \dots A_n} = \sqrt[n]{[a_{1L} a_{2L} \dots a_{nL}, a_{1U} a_{2U} \dots a_{nU}]} = [\sqrt[n]{a_{1L} a_{2L} \dots a_{nL}}, \sqrt[n]{a_{1U} a_{2U} \dots a_{nU}}], \text{ since } a_{iL} \geq 0.$$

Now, applying $AM \geq GM$ on the sets of non-negative real numbers, $\{a_{1L}, a_{2L}, \dots, a_{nL}\}$ and $\{a_{1U}, a_{2U}, \dots, a_{nU}\}$, we get

$$\frac{a_{1L} + a_{2L} + \dots + a_{nL}}{n} \geq \sqrt[n]{a_{1L} a_{2L} \dots a_{nL}} \quad \text{and} \quad \frac{a_{1U} + a_{2U} + \dots + a_{nU}}{n} \geq \sqrt[n]{a_{1U} a_{2U} \dots a_{nU}} \quad (1)$$

and the equality holds iff

$$a_{1L} = a_{2L} = \dots = a_{nL} \& a_{1U} = a_{2U} = \dots = a_{nU}. \quad (2)$$

Now, (1) implies

$$\frac{\frac{a_{1L} + a_{2L} + \dots + a_{nL}}{n} + \frac{a_{1U} + a_{2U} + \dots + a_{nU}}{n}}{2} \geq \frac{\sqrt[n]{a_{1L} a_{2L} \dots a_{nL}} + \sqrt[n]{a_{1U} a_{2U} \dots a_{nU}}}{2}.$$

Now, two cases may arise:

Case I: If

$$\frac{\frac{a_{1L} + a_{2L} + \dots + a_{nL}}{n} + \frac{a_{1U} + a_{2U} + \dots + a_{nU}}{n}}{2} \neq \frac{\sqrt[n]{a_{1L} a_{2L} \dots a_{nL}} + \sqrt[n]{a_{1U} a_{2U} \dots a_{nU}}}{2},$$

then (2) implies

$$\frac{A_1 + A_2 + \dots + A_n}{n} \geq_C^U \sqrt[n]{A_1 A_2 \dots A_n}, \text{ for all } n \in \mathbb{N}.$$

Case II: If

$$\frac{\frac{a_{1L}+a_{2L}+\dots+a_{nL}}{n} + \frac{a_{1U}+a_{2U}+\dots+a_{nU}}{n}}{2} = \frac{\sqrt[n]{a_{1L}a_{2L}\dots a_{nL}} + \sqrt[n]{a_{1U}a_{2U}\dots a_{nU}}}{2},$$

then

$$\frac{a_{1U} + a_{2U} + \dots + a_{nU}}{n} \geq \sqrt[n]{a_{1U}a_{2U}\dots a_{nU}}$$

which implies,

$$\frac{A_1 + A_2 + \dots + A_n}{n} \geq_C^U \sqrt[n]{A_1 A_2 \dots A_n}, \text{ for all } n \in \mathbb{N}.$$

Hence, combining both cases we get $\frac{A_1+A_2+\dots+A_n}{n} \geq_C^U \sqrt[n]{A_1 A_2 \dots A_n}$, for all $n \in \mathbb{N}$.
Now, the equality holds iff

$$\begin{aligned} \frac{a_{1L}+a_{2L}+\dots+a_{nL}}{n} &= \sqrt[n]{a_{1L}a_{2L}\dots a_{nL}} \text{ and } \frac{a_{1U}+a_{2U}+\dots+a_{nU}}{n} = \sqrt[n]{a_{1U}a_{2U}\dots a_{nU}} \\ \Leftrightarrow a_{1L} &= a_{2L} = \dots = a_{nL} \text{ and } a_{1U} = a_{2U} = \dots = a_{nU} \\ \Leftrightarrow [a_{1L}, a_{1U}] &= [a_{2L}, a_{2U}] = \dots = [a_{nL}, a_{nU}] \\ \Leftrightarrow A_1 &= A_2 = \dots = A_n. \end{aligned}$$

This completes the proof. ■

5 Application in Classical EOQ Model

In this section, motivated by the works of Grubbstrom [6] and Grubbsrom and Erdem [8], we study the optimal policy of the extended Harris-Wilson EOQ model in interval environment by using proposed generalised AM-GM inequality for interval numbers. Apart from the Harris-Wilson assumptions, for this model, we have considered the following assumptions and notation:

5.1 Assumptions and Notation

This model is developed under the following assumptions and notation:

5.1.1 Notation:

Notation	:	Description
$[O_L, O_U]$:	Ordering cost per order
$[h_L, h_U]$:	Carrying cost per unit per unit time
$[D_L, D_U]$:	Demand rate
$[Q_L, Q_U]$:	Ordering Quantity
T	:	Cycle length

5.1.2 Assumptions:

- (i) The demand is interval-valued.
- (ii) Holding/carrying cost and ordering cost/ set up cost are interval valued.
- (iii) The inventory system deals with single item or product.
- (iv) In the case of EOQ model, the inventory replenishment is made by giving single order.

- (v) Inventory planning/time horizon is infinite.
- (vi) Shortages are not allowed.
- (vii) Lead time is constant and known.
- (viii) Purchase price and reorder costs do not vary with the quantity ordered.

5.2 Model Formulation

The amount of order quantity at time $t = 0$ is $[Q_L, Q_U]$ units and it becomes zero at time $t = T$ due to customer's demand only. The extended EOQ model in interval environment is shown graphically in Figure 1.

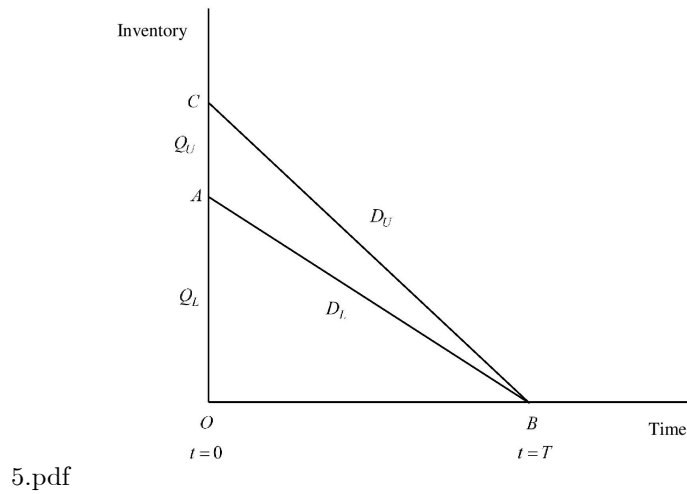


Figure 1: Variation of Inventory level at any instant.

Clearly, from the Figure 1 we get,

$$Q_L = D_L T, \quad Q_U = D_U T.$$

Therefore, the interval-valued initial order quantity is

$$[Q_L, Q_U] = [D_L T, D_U T].$$

The related inventory costs corresponding to this model are presented below:

- Ordering cost: $[O_L, O_U]$.
- Carrying cost: The bounds of interval-valued inventory carrying cost for the cycle can be calculated as follows:

$$CH_L = h_L \times \text{area of the triangle } OBA = \frac{h_L D_L T^2}{2},$$

$$CH_U = h_U \times \text{area of the triangle } OBC = \frac{h_U D_U T^2}{2}.$$

$$\text{Hence, } [HC_L, HC_U] = \left[\frac{h_L D_L T^2}{2}, \frac{h_U D_U T^2}{2} \right].$$

Hence, the total cost of this model is as follows:

$$[TC_L, TC_U] = [O_L, O_U] + [HC_L, HC_U] = \left[O_L + \frac{h_L D_L T^2}{2}, O_U + \frac{h_U D_U T^2}{2} \right].$$

Therefore, the average cost is given by

$$[AC_L(T), AC_U(T)] = \frac{[TC_L, TC_U]}{T} = \left[\frac{O_L}{T} + \frac{h_L D_L T}{2}, \frac{O_U}{T} + \frac{h_U D_U T}{2} \right].$$

5.3 Optimality Conditions

The interval valued average cost $[AC_L(T), AC_U(T)]$ can be written as:

$$\begin{aligned} [AC_L(T), AC_U(T)] &= \left[\frac{O_L}{T} + \frac{h_L D_L T}{2}, \frac{O_U}{T} + \frac{h_U D_U T}{2} \right] \\ &= \left[\frac{O_L}{T}, \frac{O_U}{T} \right] + \left[\frac{h_L D_L T}{2}, \frac{h_U D_U T}{2} \right] \\ &= \frac{2 \left[\frac{O_L}{T}, \frac{O_U}{T} \right] + [h_L D_L T, h_U D_U T]}{2}. \end{aligned}$$

Now, using generalised AM-GM inequality for interval numbers we get

$$[AC_L(T), AC_U(T)] = \frac{2 \left[\frac{O_L}{T}, \frac{O_U}{T} \right] + [h_L D_L T, h_U D_U T]}{2} \geq_C^U \sqrt{2 \left[\frac{O_L}{T}, \frac{O_U}{T} \right] [h_L D_L T, h_U D_U T]},$$

i.e.,

$$[AC_L(T), AC_U(T)] \geq_C^U \sqrt{[2h_L D_L O_L, 2h_U D_U O_U]}. \quad (3)$$

Equality holds in (3), if $2 \left[\frac{O_L}{T}, \frac{O_U}{T} \right] = [h_L D_L T, h_U D_U T]$, i.e.,

$$\frac{2O_L}{T} = h_L D_L T, \quad \frac{2O_U}{T} = h_U D_U T,$$

i.e.,

$$h_L D_L T^2 = 2O_L \text{ and } h_U D_U T^2 = 2O_U. \quad (4)$$

Then, adding both components of equations (4) we get

$$(h_L D_L + h_U D_U) T^2 = 2(O_L + O_U) \text{ i.e., } T = \sqrt{\frac{2(O_L + O_U)}{(h_L D_L + h_U D_U)}}.$$

Hence the optimal cycle length is

$$T^* = \sqrt{\frac{2(O_L + O_U)}{(h_L D_L + h_U D_U)}}. \quad (5)$$

Hence the optimal interval valued order quantity is

$$[Q_L^*, Q_U^*] = [D_L T^*, D_U T^*]. \quad (6)$$

And the optimal interval valued average cost is

$$[AC_L(T^*), AC_U(T^*)] = \left[\sqrt{2h_L D_L O_L}, \sqrt{2h_U D_U O_U} \right]. \quad (7)$$

	-20%	-10%	0%	10%	20%
T	-10.5573	-5.13167	0	4.8809	9.5445
Q_L	-10.5573	-5.13167	0	4.8809	9.5445
Q_U	-10.5573	-5.13167	0	4.8809	9.5445
AC_L	-10.5573	-5.13167	0	4.8809	9.5445
AC_U	-10.5573	-5.13167	0	4.8809	9.5445

Table 2: Sensitivity Analysis w.r.t $[O_L, O_U]$

Remark 2 If all the inventory parameters are taken as real valued i.e., $O_L = O_U = O$, $D_L = D_U = D$ and $h_L = h_U = h$, then the optimal policy for this particular case can be described as follows:

$$T^* = \sqrt{\frac{2(O+O)}{(hD+hD)}} = \sqrt{\frac{2O}{hD}} \quad (8)$$

$$[Q_L^*, Q_U^*] = [DT^*, DT^*] \leftrightarrow DT^* = \sqrt{\frac{2OD}{h}} \quad (9)$$

$$[AC_L(T^*), AC_U(T^*)] = [\sqrt{2hDO}, \sqrt{2hDO}] \leftrightarrow \sqrt{2hDO} \quad (10)$$

Which are the optimality conditions of classical EOQ model.

5.4 Numerical Illustrations

In this sub-section, the optimal policy of the proposed model is illustrated with the help of two numerical examples.

Example 1 Find the optimal cycle length, interval-valued order quantity, and the corresponding average cost of the proposed model subject to the values of interval-valued inventory parameters are considered as follows:

$$[O_L, O_U] = [300, 350], [h_L, h_U] = [3, 6], \text{ and } [D_L, D_U] = [700, 1000].$$

Solution 1 The optimal cycle length, order quantity, average cost for Example 1 is calculated by the formula (5)–(7) and their optimal values are as follows:

$$T^* = 0.401 \text{ Year}, [Q_L^*, Q_U^*] = [280.432, 401] \text{ units}, [AC_L(T^*), AC_U(T^*)] = [\$1122.497, \$2049.39].$$

Example 2 The values of the interval-valued inventory parameters considered from the crisp environment are given below to validate the optimal results in interval environment: $[O_L, O_U] = [O, O] = [325, 325]$, $[h_L, h_U] = [h, h] = [4, 4]$, and $[D_L, D_U] = [D, D] = [850, 850]$.

Solution 2 The optimal cycle length, order quantity, and average cost for Example 2 are calculated by the formula (8)–(10), and their optimal values are as follows:

$$T^* = 0.437 \text{ Year}, [Q_L^*, Q_U^*] = [371.652, 371.652] \text{ units}, [AC_L(T^*), AC_U(T^*)] = [\$1486.61, \$1486.61].$$

5.5 Sensitivity Analysis

In this section, the effects of the interval-valued ordering costs, demand rate, and holding costs on the optimal policy of the present model in Example 1 are studied by sensitivity analyses. The change of each parameter is with -20% , -10% , $+10\%$, $+20\%$ by keeping the remaining parameters fixed. Tables 2–4 show the corresponding results.

From Tables (2-4), it is observed that

	-20%	-10%	0%	10%	20%
T	11.8034	5.4093	0	-4.6537	-8.7129
Q_L	-10.5573	-5.13167	0	4.8809	9.5445
Q_U	-10.5573	-5.13167	0	4.8809	9.5445
AC_L	-10.5573	-5.13167	0	4.8809	9.5445
AC_U	-10.5573	-5.13167	0	4.8809	9.5445

Table 3: Sensitivity Analysis w.r.t $[D_L, D_U]$

	-20%	-10%	0%	10%	20%
T	11.8034	5.4093	0	-4.6537	-8.7129
Q_L	11.8034	5.4093	0	-4.6537	-8.7129
Q_U	11.8034	5.4093	0	-4.6537	-8.7129
AC_L	-10.5573	-5.13167	0	4.8809	9.5445
AC_U	-10.5573	-5.13167	0	4.8809	9.5445

Table 4: Sensitivity Analysis w.r.t $[h_L, h_U]$

- the cycle length (T) is moderately sensitive with respect to bounds of $[O_L, O_U]$, moderately sensitive with reverse effect with the change of the bounds of $[D_L, D_U]$ and $[h_L, h_U]$.
- the bounds of the interval valued order quantity $[Q_L, Q_U]$ is similarly effective as cycle length (T) with respect to the same parameters.
- the bounds of the interval valued average inventory cost $[AC_L, AC_U]$ is moderately sensitive with respect to bounds of $[O_L, O_U]$, $[D_L, D_U]$ and $[h_L, h_U]$.

From the previously mentioned discussions, it is observed that the bounds of the average inventory cost are significantly effective with change of the inventory cost factors. Thus, it is recommended the decision maker/ manager to take more care about these factors for optimal decision making.

5.6 Limitations of the Work

Though this work has discussed an alternative approach for finding the optimal policy of a classical EOQ model, its scope is limited. This approach does not apply to studying the optimal policy of any inventory model with more realistic assumptions (viz., non-linear demand rate, deterioration rate, production rate, etc.).

6 Conclusion

This work has introduced a new interval order relation which performed comparatively better/equally than the other order relations. And as an application, the generalized arithmetic mean-geometric mean (AM-GM) inequality for interval numbers is derived by the proposed interval order relation. Then, the optimal policy of classical Harris-Wilson's EOQ model in the interval environment is studied using the proposed generalized AM-GM inequality and illustrated with the help of numerical examples.

For future investigation, one may try to obtain the optimal policy of the EOQ model with shortages and the EPQ model with interval uncertainty by using this generalized AM-GM approach. Also, the same work can be extended under Type-2 interval uncertainty.

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