Blowup for the Keller-Segel System

Van Tien Nguyen
Department of Mathematics
National Taiwan University

Abstract

In this talk I will present constructive examples of blowup solutions to the Keller-Segel system in $\mathbb{R}^d$.

- L^1-critical ($d = 2$): There exist finite time blowup solutions that are of Type II with finite mass. Blowup rates are quantized according to the spectrum of a linearized operator in the self-similar setting. There is also the case of multiple collapsing blowup solutions formed by a collision of single-solutions.
- L^1-supercritical($d \geq 3$): We exhibit finite time blowup solutions that are completely unrelated to the self-similar scale, in particular, they are of Type II with finite mass. Interestingly, the radial blowup profile is linked to the traveling wave of the 1D viscous Burgers equation. There also exist solutions that blow up in finite time with infinite mass. The solution is asymptotically self-similar with a logarithmic correction to its profile for $d = 3, 4$. We found such an asymptotic profile can be either radial or completely non-radial.

The talk is based on results obtained in collaboration with Collot (Paris Cergy), Ghoul (NYU Abu Dhabi), Masmousdi (NYU), Nouaili (Paris Dauphine), Zaag (Paris Nord).