# Curvature equation with conic singularties and integrable system

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**Abstract**

Let $\left(E_\tau, dz^2\right)$, $\tau \in \mathbb{H}$ be a flat torus. We consider the follwing PDE:

$$\Delta u + e^u = \sum_{j=1}^{N} 4\pi\alpha_j \delta_{p_j} \text{ on } E_\tau \tag{1}$$

where $\delta_{p_j}$ is the dirac measure and $\alpha_j > -1$. In the literature, equation (1) is arised from conformal geometry. Indeed, (1) is equivalent to saying that the conformal metric $ds^2 = \frac{1}{2}e^u |dz|^2$ with conic singularties at $p_j$ has the Gaussian curvature 1. By classical Liouville theorem, the curvature equation is also an integrable system which yields a complex ODE (a generalization of the classical Lame equation ) and the solvability of equation (1) is equivalent to saying that the corresponding complex ODE is always apparent and has unitary monodromy. The study of the monodromy of a general complex ODE is diffi cult in general. However, recently, we also discover its relation with KdV theory. In this talk, I will talk about this deep connection and focus on the study of the complex ODE from monodromy point of view.