Covering grids with multiplicity

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Abstract

Let $S_1, S_2 \subseteq \mathbf{R}$ be two finite sets of size n, and suppose we wish to cover the points of the grid $\Gamma = S_1 \times S_2 \subseteq \mathbf{R^2}$ with as few lines as possible. It is straightforward to see that at least n lines are required. However, if our lines must avoid a given point $\vec{s_0} \in \Gamma$, then a celebrated theorem of Alon and Füredi shows that the minimum number of lines in such a cover jumps to $2(n-1)$.

In this talk, we consider the multiplicity version of this problem: in a k-cover, for some $k \in \mathbf{N}$, the lines must continue to avoid $\vec{s_0}$, but cover all other points of Γ at least k times. We shall show that the smallest k-cover of a typical grid contains $\left(\frac{3}{2} + o(1)\right) k(n-1)$ lines, improving a bound given by Ball and Serra. However, the standard grid $\Gamma_n = \{0, 1, \ldots, n-1\} \times \{0, 1, \ldots, n-1\}$ can be covered with fewer lines, and we will give nontrivial lower and upper bounds on the size of its smallest k-cover.

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