Solving the quadratic surfaces intersection problem

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Abstract

In this talk, we try to answer an open question, often referred to as the (QSIC) problem, proposed by Pólik and Terlaky in SIAM Review 2007 that: *how we can decide whether two quadratic surfaces,* $\{f = 0\}$, $\{g = 0\}$*, intersect without actually computing the intersections?* We first formulate (QSIC) into a polynomial optimization problem of degree 4, denoted by Problem (P). If the joint numerical range $\{(f(x), g(x))|x \in \mathbf{R}^n\} \subset \mathbf{R}^2$ is convex, we can compute the optimal value of (P) by the separating hyperplane theorem. Otherwise, we reduce (P) to become a quadratic optimization problem with one equality quadratic constraint and solve by $\mathcal{S}$-lemma with equality. The real difficulty comes when the two quadratic surfaces do not intersect but one approaches asymptotically to the other. It is equivalent to that the optimal value of problem (P) is not attainable, a difficult but fundamental problem which even the optimization community seldom want to handel. We resolve the feasibility issue by studying the topological separation property of the two quadratic surfaces $\{f = 0\}$, $\{g = 0\}$ and thus solve the (QSIC) problem completely.