Bifurcation lemma and its applications to the inverse eigenvalue problem

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Abstract

The inverse eigenvalue problem studies the spectral properties among a given set of matrices. For example, Colin de Verdière defined the parameter $\mu(G)$ as the largest multiplicity of λ_2 (the second smallest eigenvalue) among all weighted Laplacian matrices of G with the strong Arnold property, a non-degeneracy condition, and showed that $\mu(G) \leq 3$ if and only if G is planar. A key to this successful result is that the strong Arnold property allows us to perturb the matrix into a desired zero-nonzero pattern without changing its spectral property. In contrast, we will introduce the bifurcation lemma—a matrix with the strong property can be perturbed so that its spectral property changes slightly while its pattern stays the same—and demonstrate its impact on the inverse eigenvalue problem.