Coherent Springer theory

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Abstract

We will begin by discussing the geometric solution to the Kazhdan-Lusztig conjecture via D-modules and perverse sheaves on the flag variety, which involves identifying a category of representations with a category of constructible sheaves on a stratified space, under which one can parameterize irreducible representations by strata, and compute Ext groups in terms of cohomology. In a sense, such data gives a complete description of this finite length category.

We will then discuss Springer theory, which employs similar methods toward the character theory of a reductive group with finite coefficients $G(\mathbf{F}_q)$. The first object one encounters is the *Springer sheaf*, which knows about characters of unipotent principal series representations of $G(\mathbf{F}_q)$. A contemporary perspective sees it as being born from a categorical trace formalism applied to the 2-category of categorical representations of the group G.

Finally, we will move on to coherent Springer theory, which arises in a manner much like Springer theory, instead with applications to representations of reductive groups with coefficients in a non-Archimedian local field $G(F)$. The representation theory of $G(F)$ is not finite length, thus it is expected that any geometric parameterization of its objects should come in families, i.e. it should be a moduli stack, which brings us to algebraic geometry. To reach this realm, we pass through Langlands duality from the automorphic/representation side to the spectral/Galois side. The categories here contain coherent sheaves, in particular the coherent Springer sheaf, a sheaf on a stack of unipotent Langlands parameters closely related to the unipotent principal series representations for $G(F)$.