General Relativity: A Centennial Perspective

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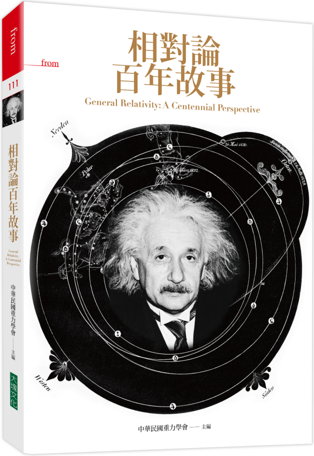
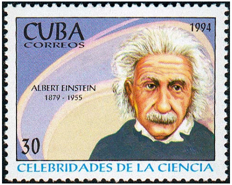
GR: A Centennial Perspective



Einstein Vs Bergson in 1922



1. 1922 April 6^{th}, Einstein & Bergson debate on ‘Time’ at Socit franaise de philosophie Paris
2. Bergson: What does it mean when the arms of clock point at 12pm?
3. Einstein: Philosophy has no play in ‘Time’
4. Is Einstein absolutely correct?
5. Bergson’s question hints at the roles of Cauchy initial surface for evolution

What is Time?

1. What is Time? Where does it come from? We all some intuitive understanding of time
2. Time is:
   - Fundamental
   - Emergent (present in semi-classical context only)
   - An illusion(Parmenides, Zeno paradox,....)

The Measure of Time

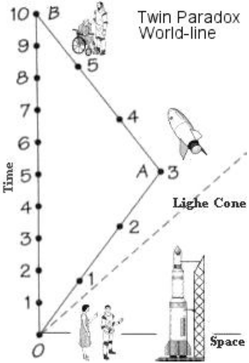
From Wikisource

The Measure of Time (1898)
by Henri Poincaré, translated by George Bruce Halsted

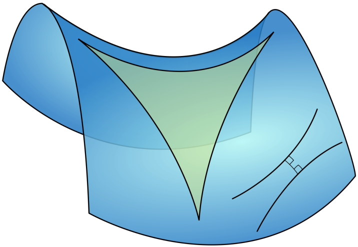
In French: *Poincaré, Henri (1898), "La mesure du temps", Revue de métaphysique et de morale* ***6****: 1-13*

If now it be supposed that another way of measuring time is adopted, the experiments on which Newton's law is founded would none the less have the same meaning. Only the enunciation of the law would be different, because it would be translated into another language; it would evidently be much less simple. So that the definition implicitly adopted by the astronomers may be summed up thus: Time should be so defined that the equations of mechanics may be as simple as possible. In other words, there is not one way of measuring time more true than another; that which is generally adopted is only more *convenient*. Of two watches, we have no right to say that the one goes true, the other wrong; we can only say that it is advantageous to conform to the indications of the first.

World *with Time & without Time*



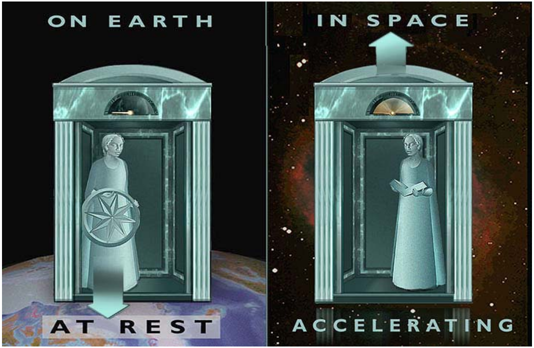
World with Time



World without Time

What is General Relativity in one sentence?

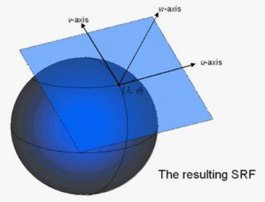
1. GR is a physical theory of curved spacetime(Universe) based on the Equivalence principle
2. What is the Equivalence principle?
   - $Inertia\ mass = Gravitational\ mass, at\ any\ one\ point$



   - Gravitational effect can be transformed away by coordinate changes at any one point

Equivalence Principle realized by curved space-time

1. Einstein asked: What type of gravity theory can be made to look locally like Special Relativity by coordinate transformations?
2. Einstein concluded: Gravitation must arises from space-time curvature (therefore, locally at any point we can have a flat tangent space)



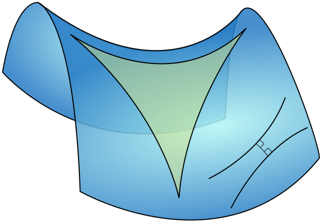
3. GR in one sentence: Space-time is semi-Reimannian, with gravity represented by its curvature & its metric satisfies Einstein's field equation

Why General Relativity is so confusing even till today?

1. Math was not sufficiently refined in 1917 to cleave apart the demands for 'no prior geometry' & geometric, coordinate independent formulation of physics
2. Einstein described both demands by a single phrase, 'general covariance'. The 'no prior geometry(frame independent)' demand actually fathered GR, but by doing so anonymously, disguised as 'general covariance', it also fathered half a century of confusion
3. Throughout his life, Einstein had never completely succeeded in response to Kretschmann(1915)'s critics that General Covariance is vacuous
4. People(include Einstein) doesn't understand difference between general covariance and symmetries of the fields in the theory & also fails in identifying the true symmetries of GR
5. More important; people can fool themselves by applying sophisticated mathematics which bypass the real problems

Mathematical achievements add further confusions

1. People confuse between covariance and frame(geometry) independence, i.e. writing Newtonian mechanics in covariant form $m\frac{d\vec{v}}{dt} = \vec{F}$(ironically, Einstein didn't believe this was possible!) doesn't mean that it is background frame independent
2. People trust mathematics more then physics; i.e. lots of effort were devoted to rewrite GR using differential geometry techniques in explicit coordinate independent differential forms



3. These developments add even more confusions; because general covariance is not equivalent to the symmetries of GR; i.e QED is covariant, but its symmetry is $U(1)$ electric charge conservation

Einstein didn't believe in Gravitational Wave(GW) in 1936

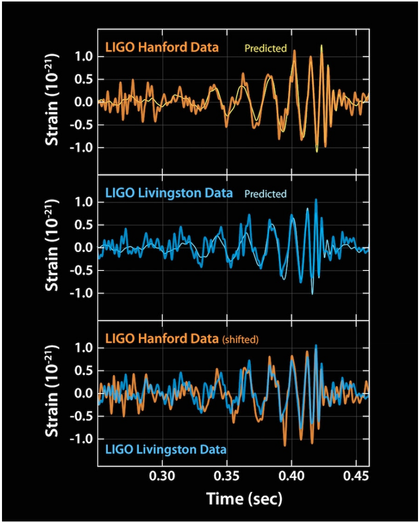
In a letter to to his friend Max Born, probably written sometime during 1936, Albert Einstein reported

> Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they had been assumed a certainty to the first approximation. This shows that the non-linear general relativistic field equations can tell us more or, rather, limit us more than we have believed up to now. (Born 1971, p. 125)

Very special features of GR:

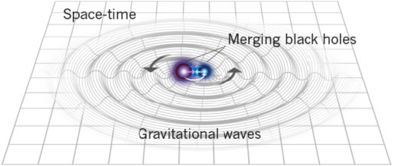
1. EOM is constrained by the field equations to guarantee its symmetry
2. Necessary to show motions in question were allowed by the same field eqt. This is more important when one considered what type of motion gave rise to radiation, i.e. who will radiate? particle in free falling or being held in observer's hand, answer is not obvious; or if BH merging following geodesic radiates or not?

LIGO's GW150914 on 14 Setp., 2016 at 09:50:45UT



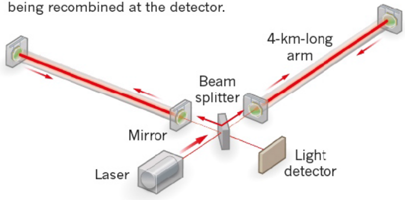
HOW LIGO CAUGHT A WAVE

The Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) has detected ripples in the fabric of space-time predicted by Einstein's general theory of relativity.

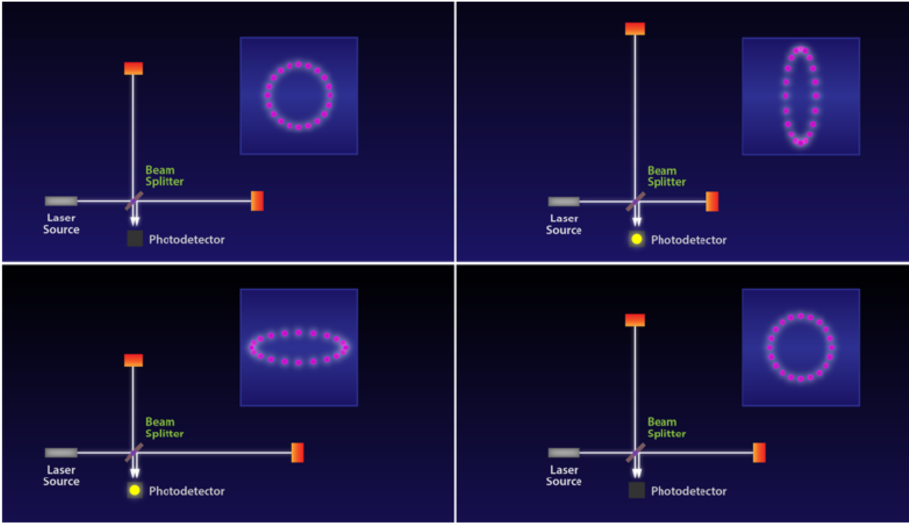


The gravitational waves were produced when two black holes — one weighing 36 solar masses and the other 29 — spiralled towards each other and merged, distorting the space-time around them in the process.

In the LIGO facility, a laser beam is split to travel down two perpendicular 4-kilometre tunnels. The beams then reflect back and forth before being recombined at the detector.

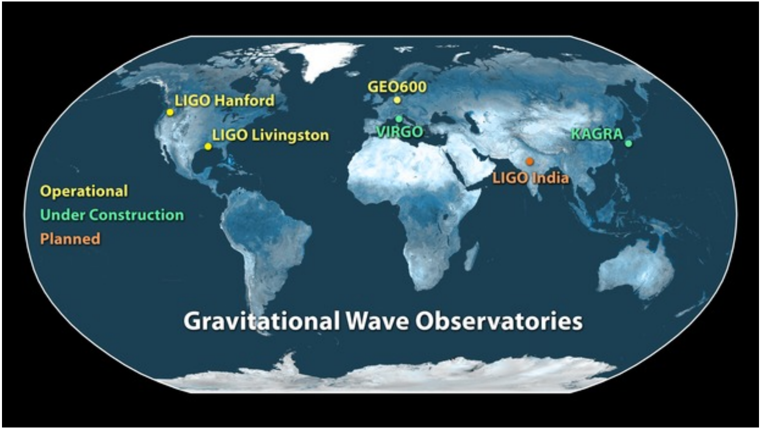


How LIGO works



Kip's visit to us

GW network world-wided



What are problems between GW & GR?

1. GW poses 3 problems: that of
   - Propagation: Expanding around Minkowski vacuum;

     $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \Rightarrow \Box h_{ij}^{TT}(\mathbf{x}) = 0$ with solution $h_{ij}^{TT}(\mathbf{x}) = \zeta_{ij} exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$

     $\Rightarrow$ Two local d.o.f. despite diffeomorphism
   - Generation: i.e. in the radiation zone $h_{ij}^{TT}(t, r, \mathbf{n}) \simeq \frac{2G}{c^4 r}\frac{\partial^2}{\partial t^2}[I_{ij}(t - \frac{r}{c})]^{TT}$
   - Detection: $\frac{\delta L}{L} \simeq h$, for $r \geqslant 6 \times 10^8$ l.y., $\frac{\omega}{2\pi} \simeq 100Hz$, $h \leqslant 10^{-21}$

     $\Rightarrow$ absorption of energy
2. These may or may not be compatible with the 4d General Covariance of Einstein's GR, i.e. $H(x) = 0$
3. Feynman(1957): 'don't be so rigorous or you will not succeed'

GW(radiation) requests Quantum theory of gravitation

Later in the conference an interesting exchange took place during the section on quantization of gravity. During Richard Feynman's presentation on the need for a quantum theory of gravity, Rosenfeld made the following remark:

> It seems to me that the question of the existence and absorption of waves is crucial for the question whether there is any meaning in quantizing gravitation. In electrodynamics the whole idea of quantization comes from the radiation field, and the only thing we know for sure how to quantize is the pure radiation field. (De Witt 1957, p. 141)

GR after LIGO's discovery of GW

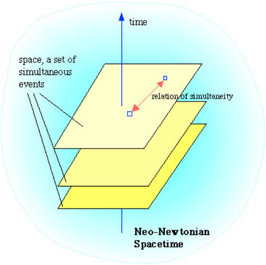
1. LIGO shows existence of GW, so it is time to confront:
   - GW is massless($< 1.2 \times 10^{-22} eV$) radiation
   - GW needs a background independent and dynamical metric explanation
   - Existence of GW radiation $\Rightarrow$ Gravity is not emergent(like em wave imply QED) and this radiation naturally requires quantization as in QED
   - GW manifests the 2 local d.o.f of GR, despite diffeomorphism invariance. If diffeomorphism invariant theory can have 2 local excitations, why shouldn't they carry local(not global or quasilocal) energy? If a gravitational wave in flat space has wavelength λ what is its momentum and energy? Doesn't the de Broglie relation and $E = h\nu$ hold?
   - Need dispersion relation; satisfies particle-wave duality to transit from QM to QFT i.e. dispersion relation $v^i = \frac{p^i c^2}{E}$; correctly captures particle-wave duality for massless photon; $E = h\nu,\ p = \frac{h}{\lambda}$ to build the corresponding QFT
2. 3dDI ITQG resolves these problems & allows generalize potential V

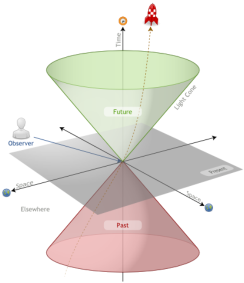
Einstein Vs Minkovski in Special Relativity

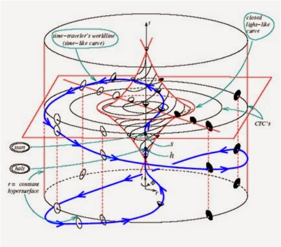
1. Einstein emphasized on algebraic properties of theory, equations remind the same form under transformations; its covariance
   - Under Lorentz transformation: $x' = \gamma(x - vt),\ t' = \gamma(t - \frac{vx}{c^2})$; equations preserved their form
2. Minkowski emphasized on geometric properties of the theory, on those geometric entities which retain unchanged behind transformations; its invariance
   - Only space-time vectors, tensors, may appear in physical equations
   - Lorentz covariance in Special Relativity is an accident (arises from homogeneity of the space-time), NOT a must of Nature(i.e. space-time may even not exist)
   - GR cannot be a generalization of Lorentz covariance
3. Fock & Synge: If relativity is physical it can't be general, if it is general it can't be relativity

Most important advantage of QM over Newton

1. Causality arises from spacetime properties in Newtonian Mechanics(time flows absolutely), in Special Relativity, it is accidentally arises from homogeneity of Minskowski space(NOT a must) & becomes ill defined in GR due to possible arbitrariness of space-time manifold







2. The lost of Causality in GR forces nature to become QM at fundamental level
3. In QM, Causality is regained through dynamical Time-orderings due to the non-commutativity natural of H at different time, i.e. $U(t, t_0) = Texp[-\frac{i}{\hbar}\int_{t_0}^{t} H(t')dt']$

QM regains Causality in the presence of gravitation

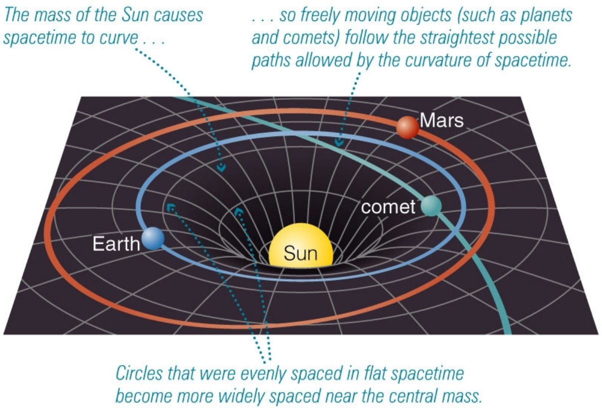
1. Newtonian mechanics is good & equipped with Causality, only drawback is 2^{nd}-order-in-time; Hamilton improved it by introducing phase spaces & dynamics now dictated by a pair of 1^{st}-order eqt
2. Gravitation causes the lost of Causality due to possible arbitrariness of space-time manifold & nature is forced to be QM in the presence of gravity at the fundamental level to recover Causality
3. This solves the biggest mystery in human civilization: replacing $\{x, p\}_{PB} = 1$ by $[x, p] = i\hbar$ is to allow non-commutativity to regain Causality in QM
4. This replacement allow matrix multiplication to replace the more restrictive differentiation; guarantees evolution can always carry out without trouble; Universe's computability is granted
5. Lagrangian path integral formalism which bypass non-commutative is therefore the worst damage to the understanding of the root of the Universe's Quantum nature
6. T-ordered evolution; $\Psi(t + \Delta t) = Texp[-\frac{i}{\hbar}\int_t^{t+\Delta t} H(t')dt']\Psi(t)$

QM is Causal, Simple, Unifying & Computable

6. 1^{st} order S-Eqt is completed; no of Q nos $=$ no of dynamical dof $+1$
7. Single Ψ unifies millions of coupled Newtonian eqts. - Real Unification
8. QM is the simplest dynamics one can imagine(just matrix multiplications), guarantees no bud can arise during the evolution of Ψ of the Universe; i.e. no need to reboot the Universe; computability guaranteed
9. QM & Schr$\ddot{o}$dinger eqt. can still be true even if there is no space-time
10. Gravitation causes the lost of Causality but regained through QM, this explains why the Schr$\ddot{o}$dinger eqt. has to be derived within gravity which is therefore universally applicable
11. 'Time' is defined through the Hamiltonian in the Schr$\ddot{o}$dinger eqt; $\therefore$ it won't fluctuate & is spacetime background independent
12. 'Time', Hamiltonian & QM are there even spacetime doesn't exists; Time is to evoke beings -things come into existence and facts become true all the time. This is one meaning of the reality of time.

Major fallacy caused by J. Wheeler I

1. Curvature tells matter to move and matter tells spacetime to curve

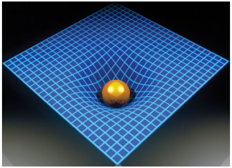


Major fallacy caused by J. Wheeler II

2. Why these eye-catching & innocent slogans create confusions:
   - It is not curvature but the Hamiltonian who is telling how everybody shall move
   - Newton's law, $Force = \dot{p} = \{p, H\}$ is still there even in GR
   - One should construct usual dynamics of $\dot{q}_{ij}$
3. Geometrization make energy and momentum ill defined
   - Energy = Force x Space
   - Momentum = Force x Time
   - If forces become curvature, energy and momentum become ill defined
4. Evolution of geometry itself is dictated by the Hamiltonian
5. Newtonian mechanics is good, only 1^{st} law become problematic due to definition of inertial frame in curved spacetime
6. Weyl curvature tensor(conformal structure) which describes real gravitational d.o.f drops out from GR(matter can't tell spacetime how to curve completely), explains why naive quantization of GR is not renormalizable(lack of conformal structure)

What is General Relativity exactly?

1. General principle of relativity (confuses with General Covariance) is no a-priori preferred metric; (Einstein said he had got rid of the last remnant of spacetime(need real care to understand)), the metric is det'd by local EOM, like electric potentials in QED



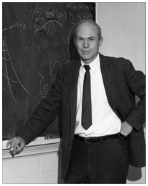
2. Central question to ask is: there are many covariant theories, what is the symmetry group of the theory? i.e. how dynamical fields are transf./relabelled under the symmetry group
3. This requires redundancy in the field components; a tragedy in modern history of QFT that the terminology 'gauge symmetry' is used instead of 'constrained theory'
4. GR is a gauge theory with dynamical d.o.f being the metric which transforms under the ∞ dim. Diff. Group; usual gauge theory is: $\prod_x G_x$ (also ∞ in dim.); $\therefore$ principle fiber bundle frameworks based on local gauge group i.e $GL(4R)$ won't work for gravity

GR is problematic(*Time problem*), ad hoc arguments

1. GR's 4-covariance has 4 symmetry generators (constraints)
   - Momentum constraint generates 3d spatial diffeomorphism invariance
   - Need one more for temporal diffeomorphism invariance; not surprising, the corresponding constraint is the Hamiltonian, $H(x) = 0$

     $\Rightarrow$ Local energy density in the theory therefore vanishes all together
2. Weyl curvature tensor(conformal structure) drops out from GR(matter can't tell spacetime how to curve completely), explains why naive quantization of GR is not renormalizable
3. Why cosmological principle based on constant 3-curvature slicing is successful? i.e. CMB is physically measured, why?
4. No way to implement Causality self-consistently i.e. Gödel Universe allows one to travel anywhere in spacetime; backward to the past(therefore QM is needed)
5. Future has yet to come

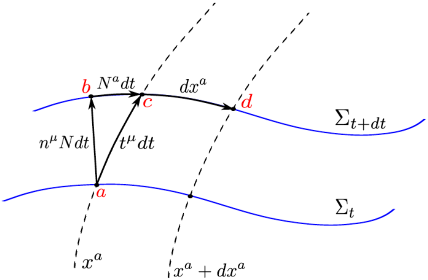
More rigorous analysis on problems with GR



1. Dirac: 4 covariance space-time is not a symmetry of GR
2. Wheeler: 3-geometry not 4-geometry; metric delivers a notion of 'simultaneity' and a common moment of a rudimentary 'time'
3. DeWitt: Time must be determined Intrinsically; DeWitt supermetric has $(-+++++)$ signature, -ive mode serves as Intrinsic Time

ADM 3+1 Decomposition: A way to see physics in GR

- 1^{st} 50 yrs of GR was wasted in finding particular solutions; reborn at initial value problem



- $ds^2 = -Ndt^2 + q_{ij}[N^i dt + N dx^i][N^j dt + N dx^j]$
- Geometrical in natural; number of points are the same

The Hamiltonian and Momentum Constraint of GR

1. First class constraints of GR
   - Hamiltonian constraint: $\mathcal{H}(x) = 2\frac{\kappa}{\sqrt{q}}(\pi^{ij}\pi_{ij} - \frac{\pi^2}{2}) - \frac{\sqrt{q}}{2\kappa}(R - 2\Lambda) = 0$
   - Momentum constraint: $\mathcal{H}_i(x) = -2\nabla_j \pi^j_i = 0$
   - These are the 1^{st} class constraints that generate gauge sym. & obey the Dirac Alg.

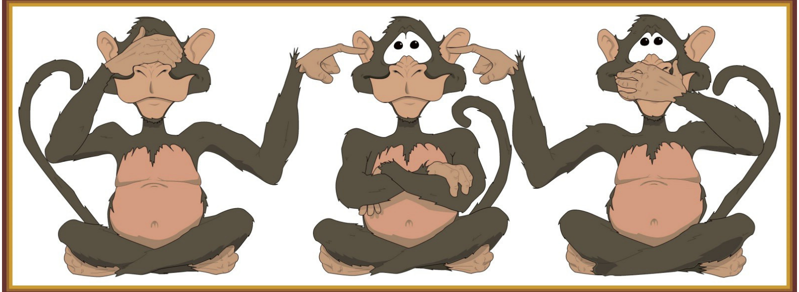
$$\{\mathcal{H}_i[N^i], \mathcal{H}_j[M^j]\} = \mathcal{H}_i[(\mathcal{L}_{\vec{N}} M)^i]$$
$$\{\mathcal{H}_i[N^i], \mathcal{H}[M]\} = \mathcal{H}[\mathcal{L}_{\vec{N}} M]$$
$$\{\mathcal{H}[N], \mathcal{H}[M]\} = \mathcal{H}_i[(g^{ij}(x))(N\partial_j M - M\partial_j N)]$$

   - Structure function(not Lie Algebra); Not algebra of 4d Diff
   - On-shell GR generates an apparent 4d Diff.
2. Momentum constraint does generate correct gauge symmetry transformations for GR
   - 3d $Diff$. Inv. Gauge Sym. $\begin{cases} \delta_{\vec{N}} q_{ij} & = \mathcal{L}_{\vec{N}} q_{ij} = \{q_{ij}, \mathcal{H}_i[N^i]\} \\ \delta_{\vec{N}} \pi^{ij} & = \mathcal{L}_{\vec{N}} \pi^{ij} = \{\pi^{ij}, \mathcal{H}_i[N^i]\} \end{cases}$

Time is confusing in GR?

Hamiltonian bears dual roles in General relativity

- Hamiltonian generates dynamical evolution, i.e. $\dot{q} = \{q, H\}$
- Hamiltonian, $H(x) = 0$ is a first-class constraint and generates Temporal Diff. Gauge Transformations, like Gauss law in QED generates $U(1)$ gauge symmetries
- But time can't be a guage
  $\Rightarrow$ Dual roles of H must be resolved; H: To be or not to be
- Hamiltonian const.'s role & problem of time can't be resolved within classical regime $\therefore$ QM is needed at fundamental level; also requires deeper insight into structures of Gauge Th.



“See no time” “Hear no time” “Speak no time”

Have to ask questions doesn't exist in Einstein's time

1. To go beyond Einstein, one has to ask correct quetions
2. Questions that didn't exist in Einstein's time
    - What is Gauge Field Theories?
    - How to gauge fermions? Weyl fermions in particular!

What is the physics of gauge field configurations?

1. Gauge fields allow one to make changes/relabelling locally at a point
2. To make changes/relabelings possible, there must exist redundant components within the field configurations
3. Since dynamics only due with real dynamical d.o.f, $\therefore$ one needs to constraints the redundant components
4. Using gauge instead of constraint terminology causes confusions
5. In QF, all symmetry/constraint operators had to be constructed from the QF, not deduced from coord. transf. $\because$ coord. are just dummies
6. Constraint is not easy to appreciate classically; quantum mechanically, it is simply that physical Q states are invariant under the relabelling of field components
7. Invariant of Q states limits the constraint operators(which generate gauge transformations) to be 1^{st}-order in canonical momentum

How to see General Covariance as gauge transformation?

1. What is Gauge transformations?
   - i.e. in QED; $\delta_{gauge} A_\mu(x) = \partial_\mu \Lambda(x)$ changes(relabel) of field variables are written in the same point and the same coordinate frame of reference
2. Under General Coordinate transformations
   - $x'^\mu = f^\mu(x^\nu)$; $g'^{\mu\nu}(x') = \frac{\partial x'^\mu}{\partial x^\alpha}\frac{\partial x'^\nu}{\partial x^\beta} g^{\alpha\beta}(x)$ not very much like Gauge Transf.!
3. However, under infinitesimal coordinate transformations; $x^\mu \to x'^\mu = x^\mu + \xi^\mu(x)$;
   - $g'^{\mu\nu}(x') = g^{\mu\nu}(x) + \xi^\nu_{,\alpha} g^{\mu\alpha}(x) + \xi^\mu_{,\alpha} g^{\alpha\nu}(x) + O\left(\xi^2\right)$
   - write field variables in the same coordinate frame of reference,

     $$\Rightarrow g'^{\mu\nu}(x') = g'^{\mu\nu}(x^\alpha + \xi^\alpha(x)) = g'^{\mu\nu}(x) + g^{\mu\nu}_{,\alpha}\xi^\alpha + O\left(\xi^2\right)$$
   - $\delta_{Diff} g^{\mu\nu}(x) = g'^{\mu\nu}(x) - g^{\mu\nu}(x) = \xi^\nu_{,\alpha} g^{\mu\alpha}(x) + \xi^\mu_{,\alpha} g^{\alpha\nu}(x) - g^{\mu\nu}_{,\alpha}\xi^\alpha$
   - a mixture of both coordinate and field transformations
4. $\therefore$ Diffeomorphism is a GT(at same point.): $\boxed{\delta_{Diff} g^{\mu\nu}(x) = \xi^{\mu;\nu} + \xi^{\mu;\nu} = \mathcal{L}_\xi g^{\mu\nu}(x)}$
5. But what is the corresponding symmetry generator(constraint) in the context of QFT?

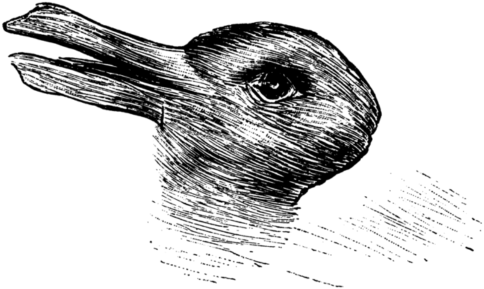
Meaning of Constraints & Gauge Invariance

1. Gauge Inv. means:
   - $\Psi[A_{ia} + \delta_{gauge} A_{ia}] = \Psi[A_{ia}]$
   - Constraint annihilates Gauge Inv. states, $\mathcal{C}\Psi[A_{ia}] = 0$
   - Constraints generate Gauge transf. and be 1^{st} order in canonical mom.
2. GR w.r.t spatial diff. is in fact a gauge theory with metric as gauge variable

$$\Psi[q_{ij} + \delta_{gauge} q_{ij}] = \Psi[q_{ij}] + \cancel{\int (\delta_{gauge} q_{ij}(x)) \frac{\delta \Psi}{\delta q_{ij}(x)}} d^3x = \Psi[q_{ij}]$$

   - Since $\delta_{gauge} q_{ij} = \mathcal{L}_{\vec{N}} q_{ij}; \tilde{\pi}^{ij} = \frac{\hbar}{i} \frac{\delta}{\delta q_{ij}}$ therefore $\int N^i H_i \, d^3x \Psi[q_{ij}] = 0$
   - Not possible for constraints quadratic in mom. such as Hamiltonian constraint H
   - Despite that 3d Diff. group is not a ∞ tensor product at every point as in ordinary gauge theory, GR is indeed a gauge theory but not of the principle fiber type

Welche Thiere gleichen ein-
ander am meiſten?



Kaninchen und Ente.

- Hegel: Things which look different are really the same
- Wittgenstein: Things which look the same are really different

GR Hamiltonian constraint is not a gauge sym. generator

1. Interpretation of changes(gauge) generated by $H(x)$ is only true on shell
   - $\delta_N q_{ij} = \{q_{ij}, H[N]\} = \frac{\kappa}{\sqrt{q}}(q_{ik}q_{jl} + q_{il}q_{jk} - q_{ij}q_{kl})\pi^{kl} = \mathcal{L}_{N\vec{N}} q_{ij}$ [modulo EOM]
   - $\delta_N \pi^{ij} = \{\pi^{ij}, H[N]\} = \frac{N}{2} q^{ij} H - N\sqrt{q}(q^{ki}q^{lj} - q^{ij}q^{kl})R_{kl} + \mathcal{L}_{N\vec{N}}\pi^{ij}$
   - $H = 0$ is a constraint but doesn't generate gauge sy. instead, real Diff.(evolution)
2. In the quantum context, GR does not possess 4d Diffeomorphism inv.
3. Physics of GR can't be fully appreciated within classical regime
4. 4d spacetime covariance CANNOT be the full sym. of Gravitation & is only apparently true strictly at classical level; true sym. of Gravitation is 3dDI

Diffeomorphism gauge & Yang-Mills gauge structures

Table 1. Comparison of the two different gauge structures

	Diffeomorphism Gauge Structures	Yang-Mills Gauge Structures
Basic Variables	Spatial metric tensor q_{ij}	Gauge connection A_{ia}
Symmetry Generators	$H_i(\mathbf{x}) = -2q_{ik}\nabla_j\pi^{jk}(\mathbf{x})(=0)$	$G^a(\mathbf{x}) = \nabla_i\pi^{ia}(\mathbf{x})(=0)$
Gauge transformation	$[q_{ij}(\mathbf{x}), H_k[N^k]] = \mathcal{L}_{\vec{N}}q_{ij}(\mathbf{x})$; $H_i[N^i] = \int N^i H_i d^3\mathbf{x}$	$[A_{ia}(\mathbf{x}), G^b[\eta_b]] = -\nabla_i\eta_a(\mathbf{x})G^b[\eta_b]$; $G^b[\eta_b] = \int \eta_b G^b d^3\mathbf{x}$
Commutation Relations	$[H_i(\mathbf{x}), H_j(\mathbf{y})]$ $= H_j(\mathbf{x})\partial_i\delta(\mathbf{x}-\mathbf{y}) + H_i(\mathbf{y})\partial_j\delta(\mathbf{x}-\mathbf{y})$	$[G^a(\mathbf{x}), G^b(\mathbf{y})]$ $= if^{ab}{}_c G^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})$
Potentials	$V \sim [\frac{\delta exp(CS)}{\delta q_{ij}}]^2$	$V \sim [\frac{\delta exp(CS)}{\delta A_{ia}}]^2$
Locality & Dimension	Not product of identical group(i.e. $SL(3R)$) at each spatial point of base manifold i.e. not of principle fibre structure	Infinite tensor product group $\prod_x G$; G=finite dimensional Lie group =usually referred to as the 'gauge group'

Intrinsic Time extracted from 3-metric

1. Dispensing Hamiltonian as gauge generator leads to extra pair of d.o.f, $\therefore$ need extra care

2. Decompositing $(q_{ij}, \tilde{\pi}^{ij})$ into conjugate pairs: $(\bar{q}_{ij}, \bar{\pi}^{ij})$, $(\ln q^{\frac{1}{3}}, \pi)$; & identifying $\ln q(x)$ as intrinsic time, $\pi = q_{ij}\tilde{\pi}^{ij}$ being the energy function(Dirac's extended phase space)

3. ADM Hamiltonian constraint($\beta^2 = \frac{1}{6}$ for GR) will deliver dynamical eqt.

   - $H(x) = -qR + \bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} - \beta^2\pi^2 = 0$

   - Dynamical eqt & local Hamiltonian density derived from above factorization is:

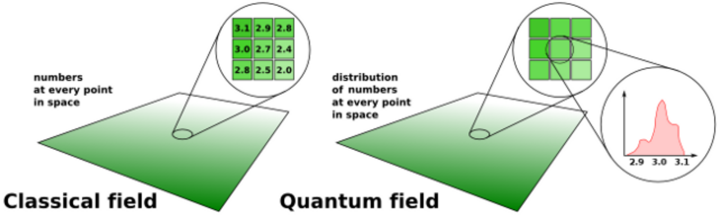
     $$-\pi = \frac{\bar{H}(x)}{\beta} = \frac{1}{\beta}\sqrt{\bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} - qR} \rightarrow \frac{1}{\beta}\sqrt{\bar{q}_{ik}\bar{q}_{jl}\bar{\pi}^{ij}\bar{\pi}^{kl} - V}$$

   - This local Hamiltonian density correctly captures the dispersion relation $v = \frac{p}{E}$ & particle-wave duality with upper limit in speed; readily for carrying out quantization

   - This derives the Schr$\ddot{o}$dinger eqt. upon quantization: $i\hbar\frac{\delta\Psi}{\delta \ln q} = \frac{\bar{H}(\hat{\bar{\pi}}^{ij};q_{ij})}{3\beta}\Psi$, although intrinsic time $\delta \ln q(x)$ in it is multi-finger & non-integrable

Properties of Quantum Fields & Gauge d.o.f

1. Quantum Fields are distributions with spatial and time dependence



2. In QF Theories everything(observables) has to be built from QFs
3. Even space and time has to be built from QF(which is just the metric QF field)
4. 'Time' is a single parameter, how to collapse a distribution of QFs into a single paramter?
5. $\Rightarrow$ there must be some redundancies in the QF & have to be get rid of from the theory
6. This requires the QF are gauge fields(some components are redundancies, $\therefore$ allows collapse into single parameter and won't fluctuate after collapse due to gauge principle)

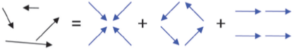
   $\Rightarrow$ the world has to be gauged to have 'Time'

Hodge Decomposition of intrinsic time $\delta \ln q^{\frac{1}{3}}$

1. $\delta \ln q^{\frac{1}{3}}(x)$ is a Tomonaga-Schwinger time; the derived Schrödinger eqt. is not integrable
   - Needs to collapse into single parameter δT without fluctuation

2. Hodge decomposition: $\delta \ln q^{\frac{1}{3}} = \delta T + \nabla_i \delta S^i \propto \frac{\delta V}{V}$; $V\ being\ Volume\ of\ the\ Universe$

$$\xi = \nabla u + \nabla \times \mathbf{v} + \mathbf{h}$$



   - The root of a gauged world - Universe has to be locally gauged to have time
   - The 3DDI S-eqt becomes: $\boxed{i\hbar \frac{\delta \Psi}{\delta T} = \mathcal{H}_{Phys.} \Psi}$ with $\mathcal{H}_{Phys} := \int \frac{\bar{H}(x)}{\beta} d^3x$
   - $[\mathcal{H}_{\rm Phys}(T), \mathcal{H}_{\rm Phys}(T')] \neq 0 \Rightarrow$ 3DDI 'time'-ordering & causality emerges
   - $\Psi[[q_{ij}(h)] \in {}^{(3)}\mathcal{G}]$ are GI states replace notions of events in Special Relativity

Correct variable for quantum gravity: Momentric variables

1. $\bar{\pi}^{ij}$ doesn't preserve positivity of $\bar{q}_{ij}$ & difficult to implement as self-adjoint operator

2. NEW traceless momentric: $\bar{\pi}^i_j = \bar{E}^i_{j(mn)}\tilde{\pi}^{mn}$; $\bar{E}^i_{j(mn)} = \frac{1}{2}\left(\delta^i_m \bar{q}_{jn} + \delta^i_n \bar{q}_{jm}\right) - \frac{1}{3}\delta^i_j \bar{q}_{mn}$

$$\begin{cases} [\bar{q}_{ij}(x), \bar{q}_{kl}(y)] &= 0 \\ [\bar{q}_{ij}(x), \hat{\bar{\pi}}^k_l(y)] &= i\hbar \bar{E}^k_{l(ij)}\delta(x-y); \\ [\hat{\bar{\pi}}^i_j(x), \hat{\bar{\pi}}^k_l(y)] &= \frac{i\hbar}{2}\left(\delta^k_j \hat{\bar{\pi}}^i_l - \delta^i_l \hat{\bar{\pi}}^k_j\right)\delta(x-y) \end{cases}$$

3. Quantum mechanically, $\hat{\bar{\pi}}^i_j(x)$ can be explicitly realized in the metric rep. by

$$\hat{\bar{\pi}}^i_j(x) = \frac{\hbar}{i}\bar{E}^i_{j(mn)}(x)\frac{\delta}{\delta\bar{q}_{mn}(x)} = \frac{\hbar}{i}\frac{\delta}{\delta\bar{q}_{mn}(x)}\bar{E}^i_{j(mn)}(x) = \hat{\bar{\pi}}^{\dagger i}_j(x)$$

which are self-adjoint on account of $[\frac{\delta}{\delta\bar{q}_{mn}(x)}, \bar{E}^i_{j(mn)}(x)] = 0$

Free Theory of QG characterized by $SU(3)$

1. $\bar{\pi}^i_j(x)$ generate $SL(3,R)$ positivity preserving & unimodularity transf. of $\bar{q}_{ij}$ which

   $\Rightarrow$ free vacuum has extra scaling sym. of unimodularity transf.

2. $\bar{\pi}^i_j(x)$ by themselves generate at each spatial point an $SU(3)$ algebra,

   $T^A(x) = \frac{1}{\hbar\delta(0)}(\lambda^A)^j_i \hat{\bar{\pi}}^i_j(x)$; $[T^A(x), T^B(y)] = i f^{AB}{}_C T^C \frac{\delta(x-y)}{\delta(0)}$

3. The free theory of Laplacian operator is characterized by $SU(3)$ Casimir invariants

$$\frac{\hbar^2\delta^2(0)}{2} T^A T^A = \hbar^2 \nabla^2_{\bar{q}} = \hat{\bar{\pi}}^{i\dagger}_j \hat{\bar{\pi}}^j_i = \hat{\bar{\pi}}^i_j \hat{\bar{\pi}}^j_i = \hat{\bar{\pi}}^{ij} \hat{\bar{G}}_{ijkl} \hat{\bar{\pi}}^{kl}$$

Spectrum of the free theory

1. Free spectrum labeled by e.v. of the complete commuting set at each spatial point comprising the two Casimirs and its Cartan subalgebra T^3, T^8, and $I = \sum_{B=1}^{3} T^B T^B$

2. Functional differentiation of $\hat{\bar{\pi}}^i_j$ on Ψ now traded for group generators on $SU(3)$ states

$$\frac{\hbar}{i}(\lambda^A)^i_j \bar{E}^j_{i(mn)} \frac{\delta}{\delta \bar{q}_{mn}(x)} \langle \bar{q}_{kl}| \prod_y |l^2, C, I, m_3, m_8\rangle_y = \frac{\hbar\delta(0)}{2} \langle \bar{q}_{kl}| T^A(x) \prod_y |l^2, C, I, m_3, m_8\rangle_y$$

3. Free theory's $SU(3)$ singlet ground state with zero energy, corresponds to $l^2 = 0 \,\forall\, x$, $\hat{\bar{\pi}}^i_j(x)|0\rangle = 0$; is also 3dDI because $-2\nabla_j \hat{\bar{\pi}}^j_i$ generates spatial diffeomorphisms of $\bar{q}_{ij}$

Emergence of Einstein-Hilbert Gravity

1. $\bar{H} = \sqrt{\hat{Q}^{\dagger i}_j \hat{Q}^j_i} = \sqrt{\hat{\bar{\pi}}^{\dagger j}_i \hat{\bar{\pi}}^i_j + \hbar^2 (g\tilde{C}^i_j + b\sqrt{q}\bar{R}^i_j)(g\tilde{C}^j_i + b\sqrt{q}\bar{R}^j_i) + [\hat{\bar{\pi}}^i_j, ib\hbar\sqrt{q}\bar{R}^j_i]}$
   - $\hat{Q}^i_j = e^{W_T}\hat{\bar{\pi}}^i_j e^{-W_T} = \frac{\hbar}{i}\bar{E}^i_{j(mn)}[\frac{\delta}{\delta\bar{q}_{mn}} - \frac{\delta W_T}{\delta\bar{q}_{mn}}] = \frac{\hbar}{i}\bar{E}^i_{j(mn)}\frac{\delta}{\delta\bar{q}_{mn}} + ib\hbar\sqrt{q}\bar{R}^i_j + ig\hbar\tilde{C}^i_j$
   - $W_T = \frac{g}{4}\int\tilde{\epsilon}^{ijk}(\bar{\Gamma}^l_{im}\partial_j\bar{\Gamma}^m_{kl} + \frac{2}{3}\bar{\Gamma}^l_{im}\bar{\Gamma}^m_{jn}\bar{\Gamma}^n_{kl})\, d^3x + b\int\sqrt{q}Rd^3x$ is the Chern-Simon action
2. Zero Point Energy: $[\hat{\bar{\pi}}^i_j, ib\sqrt{q}\hbar\bar{R}^j_i] = -\frac{5}{12}b\hbar^2\delta(0)\sqrt{q}(5R - \frac{9}{\epsilon})$ incorporates the E-H term, means the simple Hamitlonian density, $\sqrt{\hat{Q}^{\dagger i}_j \hat{Q}^j_i}$ already contains Einstein's GR with Λ
3. Cotten-York tensor term preponderance at early times, Einstein's GR dominates at low curvature and long wavelengths in a theory in which '4d symmetry is not a fundamental property of the physical world'

Dimensionless Fundamental Theory

1. $\delta(0)$ which denotes the 3d coincidence limit, $\lim_{x\to y}\delta(x-y)$, was left untouched, with the understanding that it can be regularized
2. The underlying $SU(3)$ structure provides regularization of the theory

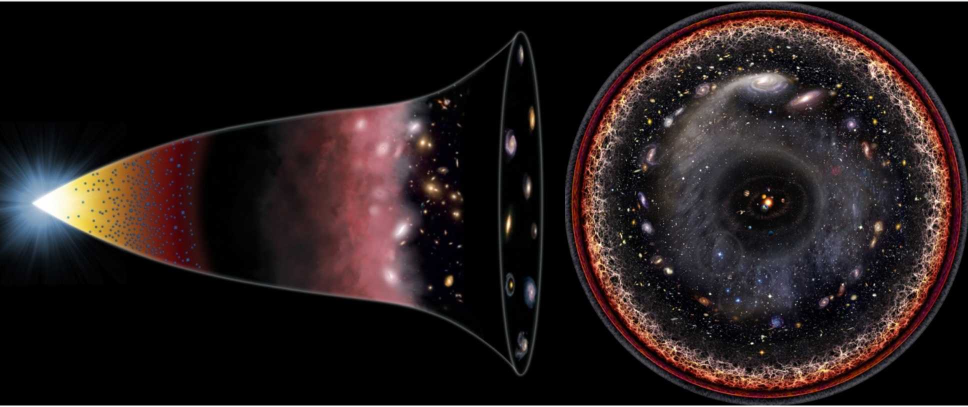
$$H_{Phys} = \hbar\int\sqrt{\left(e^{-W_T}T^A(x)e^{W_T}\right)\left(e^{W_T}T^A(x)e^{-W_T}\right)}\,\frac{\delta(0)}{\sqrt{2}\beta}d^3x;$$

- $\frac{\delta(0)}{\beta}d^3x$ is dimensionless div. to be absorbed by renormalization of β
- Cancelation of $\hbar$ on both sides of S-eqt.,i.e., $i\frac{\delta\Psi}{\delta T} = H'_{\rm Phys}\Psi$, our universe is described by a fundamental eqt with dimensionless Hamiltonian and intrinsic time
- What is paramount to causality is not the actual dimension of time but the sequence and ordering in time, generated by the non-commutative ordering of $H_{\rm Phys}(T)$. Even $\hbar$ will continue to leave its imprints in physics in conversion factor between $SU(3)$ generators T^A and the momentric, unification of gravitation and QM comes with the demotion of its elementary significance. With the dimensionless fundamental variables, the CR are
$[\bar{q}_{ij}(x), T^A(y)] = \frac{i}{2}((\lambda^A)^k_i\bar{q}_{kj} + (\lambda^A)^k_j\bar{q}_{ki}))\frac{\delta(x-y)}{\delta(0)}$; $[T^A(x), T^B(y)] = if^{AB}{}_C T^C\frac{\delta(x-y)}{\delta(0)}$
- Quantum essence is embodied in the noncommutativity, but Planck const. is absent
- Converting from T^A to extrinsic curvature K_{ij}, introduces space-time and κ

Four Pillars of whys, that make the Universe as it is

1. Causality: Foundation of order $\rightarrow$ allow nature being accessible to human minds
   - Gravity forces $\{,\}_{P.B.} \rightarrow$ to be replaced by [,] to regain Causality
   - Frame independent upper limit in speed
2. Unitary: Information preserving, none will be left behind $\rightarrow$ Universe to evolve as a whole
3. Computability: Fundamental dimensionless Schr$\ddot{o}$dinger eqt. unifies everything into a single Gauge Inv. state $\rightarrow$ $\hbar \& \kappa$ are converting quantities
   - $\{,\}_{P.B.}$ replaced by [,], replace differentiations by algebraic matrix multiplications $\rightarrow$ evolution & computability; needs no reboot
   - Gauged $\rightarrow$ to get rid of the ∞ number of d.o.f in the metric field to get down to only one d.o.f for time to exist; Time-ordered sequence is Diffeomorphism Invariant
   - Completeness $\rightarrow$ Allow a single 1^{st} order in time Schr$\ddot{o}$dinger equation to develop into trillions pairs of classical H-J equations $\rightarrow$ correspondence between q and c numbers $\rightarrow$ number of dynamical dof $=$ number of integration $+$ 1 overall constant
   - Integrability & Complexity $\rightarrow$ Unique initial state; only ONE Time & flows in the direction of entropy
4. Renormalizability: Allow point-like Fields & Geometry to play self-consistent roles
   - Point like fields(no finer structure) at fixed point make Geometrodynamics possible
   - Small/large Universe has same number of points $\rightarrow$ Universe is Geometrodynamical

Meaning of Time - A physicist's response to Bergson



Time is to evoke existence: Things come into beings and facts become true all the time & therefore flows in the same direction as entropy

Meaning requests the reality of time