1. (20 points)

(a) (10 points) Use definition (do not use Theorem 3.33) to show that the Cantor-Lebesgue function \( f(x) : [0, 1] \to [0, 1] \) is not a Lipschitz continuous function.

(b) (10 points) Show that the Cantor-Lebesgue function \( f(x) : [0, 1] \to [0, 1] \) satisfies the following

\[
|f(x) - f(y)| \leq 2|x - y|^{\alpha}, \quad \forall x, y \in [0, 1]
\]

where \( \alpha \in (0, 1) \) is a constant given by \( \alpha = \log 2 / \log 3 \). (Hint: Use the fact that if \( x, y \in [0, 1] \) with \( |x - y| \leq 3^{-k} \) for some \( k \in \mathbb{N} \), then the difference \( |f(x) - f(y)| \) is at most \( 2^{-k} \). For arbitrary \( x, y \in [0, 1] \) one can choose an unique \( k \in \mathbb{N} \) such that \( 3^{-k-1} < |x - y| \leq 3^{-k} \), which implies \( |f(x) - f(y)| \leq 2^{-k} \). Rewrite the estimate without involving \( k \).)

2. (10 points) Do Exercise 20 in p. 48.


4. (10 points) Do Exercise 23 in p. 49.