Homework #1, due Mar 10

Objectives: (1) Review Taylor series and application to finite difference and quadrature rules. (2) To illustrate basic numerical ideas. (3) To practice basic programming.

Paper and pen assignments

(1) The numerical solutions computed using 100, 200 and 400 grid points are 1.4663, 1.5861 and 1.6160 respectively. What do you think is the order of accuracy of this method?

(2) An "X-ple precision" arithmetics stores a floating point number in binary expression

$$\pm .d_1 d_2 \cdots d_s \times 2^e$$

with $s = 33$, $-509 \leq e \leq 512$. How many "bits" does it take to store such a floating point number? How large is the roundoff error for floating number representation?

(3) **(a)** Derive a 2nd order finite difference for u'' using centered difference. The error term is called the local truncation error. Keep the leading term of the error for use in part (b).

(b) Derive a 4th order finite difference discretization for u' and u''. (Hint: use a combination of grid size h and $2h$, similar to Richardson extrapolation)

Programming assignments

(4) Write a short program that displays the 3D plot of $z = x^2 - y^2$ over $[-2, 2] \times [-2.5, 2.5]$. Put your name (real name, school number or nick name) somewhere on the graph. You can use ANY plotting software. The relevant commands in matlab are `mesh`, `xlabel`, `title`, etc.

(5) We want to develop a efficient method for computing integrals like

$$\int_0^1 \sqrt{1-(1-x)^2}dx$$

In fact, we know the exact solution is $\frac{\pi}{4}$.

(a) Show that

$$\int_{x-h}^{x+h} f(t)dt - h(\frac{1}{3}f(x-h) + \frac{4}{3}f(x) + \frac{1}{3}f(x+h)) = -\frac{h^5}{90}f^{(4)}(x) + O(h^6)$$

(In fact it can be shown (non-trivially) that the right hand side is equal to $-\frac{h^5}{90}f^{(4)}(\xi)$ for some ξ). Give a rough estimate on how many grid points it takes to evaluate the integral above using Midpoint rule and Simpson's rule directly in order to get 10 correct digits. Verify your estimates numerically.

(b) Propose a better way (say, within 1,000 grid points) of computing the integral and implement it. This method should only rely on your knowledge of the behavior (regularity) and NOT on the exact form (i.e. analytically integrable). of the integrand. For example, your method should be applicable to computing, say,

$$\int_0^1 \sqrt{e^x - 1}dx$$

As an alternative, you may use an existing package, if you can find one. You'll have to explain why your package does the job.