2003. 动机请caculus作業解答

Homework 6 (P.1)

§6.1

#46. $y = t^2 e^{2/t}$, find dy/dt

By chain rule, $\frac{dy}{dt} = 2t e^{2/t} + t^2 e^{2/t} \cdot (-\frac{2}{t^2})$

$= 2(t-1) e^{2/t}$ #

#48. $y = \frac{e^{-x}}{e^x+1}$, (自証).

$\Rightarrow y' = -\frac{e^{-x}+2}{(e^x+1)^2}$ #

#60. $y = x^2 e^{-x}$, $x \in [-1, 2]$

$y' = 2xe^{-x} - x^2 e^{-x} = e^{-x}(2x - x^2)$

$\Rightarrow$ critical points are $x = 0, 2$.

$\Rightarrow y(-1) = e$, $y(0) = 0$, $y(2) = \frac{4}{e^2}$

$\therefore$ Abso. max is e at $x=-1$, abso. min. is 0 at $x=0$ #

#66. $g(x) = e^{x/\sqrt{x^4+1}}$

$g'(x) = e^{x/\sqrt{x^4+1}} \cdot \frac{1-x^4}{(x^4+1)\sqrt{x^4+1}}$

⋮ (自証)

abso. max. is $e^{1/\sqrt{2}}$ at $x=1$

" min " $e^{-1/\sqrt{2}}$ " $x=-1$ #

§6.2.

#28(c) $f(x) = 2x^2$, $x \geq 0$, $a = 5$.

$df/dx|_{x=a} = 4x|_{x=5} = 20$ #

$df^{-1}/dx|_{x=a} = \frac{1}{\left(\frac{df}{dx}\right)_{x=a}} = \frac{1}{20}$ #

§6.2 H.W.6 (P.2)

#32. $f(x) = x^2 - 4x - 5$, $x > 2$

$$df^{-1}/dx\,|_{x=5} = \frac{1}{(df/dx)_{x=5}} = \frac{1}{2x-4}\Big|_{x=5} = \frac{1}{6}$$ #

#45 $f \circ g$ in 1-1 (one-to-one)

∵ For x_1, x_2 lie in the domain of g, such that $x_1 \neq x_2$

$\Rightarrow g(x_1) \neq g(x_2)$

∵ the range of g lies in the domain of f, and

f in 1-1.

∴ $f \circ g(x_1) \neq f \circ g(x_2)$

Hence $f \circ g$ in 1-1. #

#46. "Yes",

Suppose "No", then there exists a function g such that

for some $x_1 \neq x_2$ in the domain of g

$\Rightarrow g(x_1) = g(x_2)$

let $g(x_1) = y_1$, $g(x_2) = y_2 \Rightarrow y_1 = y_2$

$\Rightarrow f(y_1) = f(y_2)$

$\Rightarrow f \circ g(x_1) = f \circ g(x_2)$, this contradict to $f \circ g$ in 1-1. #

§6.3.

#31. $y = \frac{\ln t}{t}$, find $\frac{dy}{dt}$.

$$dy/dt = \frac{t \cdot \frac{1}{t} - 1 \cdot \ln t}{t^2} = \frac{1 - \ln t}{t^2}$$ #

#32 ~ #40. (略)

#46. $y = (\tan\theta)\sqrt{2\theta+1}$, find y'

$$\ln y = \ln \tan\theta + \frac{1}{2}\ln(2\theta+1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{d\theta} = \frac{\sec^2\theta}{\tan\theta} + \frac{1}{2}\frac{2}{2\theta+1}$$

$$\therefore \frac{dy}{d\theta} = \sec^2\theta\sqrt{2\theta+1} + \frac{\tan\theta}{\sqrt{2\theta+1}}$$ #

#48. 略

#50 略.

H.W.6. (P.3)

$e + 4^{-2\log_4(x)} = \frac{1}{x}\log_{10}(100)$

$+ 4^{-2\frac{\ln x}{\ln 4}} = \frac{1}{x}\log_{10}10^2$

$+ 4^{-2\frac{\ln x}{\ln 4}} = \frac{2}{x} \Rightarrow 4^{-2\ln x/\ln 4} = \frac{2}{x} - 1$

$\to -2\frac{\ln x}{\ln 4}\cdot\ln 4 = \ln\left(\frac{2}{x}-1\right) \Rightarrow \ln x^{-2} = \ln\left(\frac{2}{x}-1\right)$

$\Rightarrow x^{-2} = \frac{2}{x} - 1 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$ #

$\left(\frac{x+1}{x-1}\right)^{\ln 3}$), find $\frac{dy}{dx}$

$\ln\left(\left(\frac{x+1}{x-1}\right)^{\ln 3}\right) = \ln\left(\frac{x+1}{x-1}\right) = \ln(x+1) - \ln(x-1)$

$\frac{1}{x+1} - \frac{1}{x-1} = \frac{-2}{(x+1)(x-1)}$ #

$\lim_{t\to 0}\frac{\pi^t - 1}{t}$

π^t, then the linearization of $y(t)$ at $t=0$ is

$L = f'(0)(t-0) + f(0) = t\cdot\ln\pi + 1$

$f(t) \approx t\ln\pi + 1$ as $t\to 0$.

$\frac{\pi^t - 1}{t} \approx \frac{(t\ln\pi + 1) - 1}{t} = \ln\pi$ as $t\to 0$. #

$\frac{(0.5)^\theta - 1}{\theta} = \ln 0.5$ by (a) #

$(x+1)^x$, find dy/dx

$(1)\ y = e^{x\cdot\ln(x+1)} \Rightarrow \ln y = x\ln(x+1)$

$\Rightarrow \frac{1}{y}(y') = \ln(x+1) + \frac{x}{x+1} \Rightarrow y' = (x+1)^x\ln(x+1) + \frac{x\cdot(x+1)^x}{x+1}$

$= (x+1)^x\left(\ln(x+1) + \frac{x}{x+1}\right)$ #

(四題) #

H.W. 6. (P.4)

§ 6.6

#18 Find $\lim_{t\to 0} \frac{t\sin t}{1-\cos t}$

$$\lim_{t\to 0} \frac{t\sin t}{1-\cos t} \overset{\text{l'Hopital}}{=\!=} \lim_{t\to 0} \frac{\sin t + t\cos t}{\sin t} \overset{\text{l'Hopital}}{=\!=} \lim_{t\to 0} \frac{\cos t + \cos t - t\sin t}{\cos t} = 2 \quad \#$$

#22, #28, #29. 略

#52. a) $\lim_{x\to -} \frac{x^2-2x}{x^2-\sin x} = \lim_{x\to 0} \frac{2x-2}{2x-\cos x} \not= \lim \frac{2}{2+\sin x} = \frac{2}{2+0} = 1$ ——(X)

$\because \lim_{x\to 0} \frac{2x-2}{2x-\cos x} = 2$ 不可再用 l'Hopital's rule.

$\therefore$ (b) is correct. #

#54 $f(x) = \begin{cases} x+2 & x\neq 0 \\ 0 & x=0 \end{cases}$, $g(x) = \begin{cases} x+1 & x\neq 0 \\ 0 & x=0 \end{cases}$

Clearly, $f'(x) = 1$ for $x\neq 0$,

$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x} = \lim_{x\to 0} \frac{x+2-0}{x}$ $\therefore f'(0)$ is undefined.

Similarly, $g'(x) = \begin{cases} 1 & x\neq 0 \\ \text{undefined} & x=0 \end{cases}$

$\Rightarrow \lim_{x\to -} \frac{f'}{g'} = \lim_{x\to 0} \frac{1}{1} = 1$ #

$\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to -} \frac{x+2}{x+1} = 2$ #

It doesn't contradict l'Hopital's rule. Since $\lim \frac{f}{g}$ is not of the form $\frac{0}{0}$, or $\frac{\infty}{\infty}$ as in #52. #

#55. $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{9x-3\sin 3x}{5x^3} = \lim_{x\to 0} \frac{9-9\cos 3x}{15x^2} = \lim_{x\to 0} \frac{27\sin 3x}{30x}$

$$= \lim_{x\to 0} \frac{81\cos 3x}{30} = \frac{27}{10}$$

$\therefore$ Let $c = \frac{27}{10}$, then f is contin. at $x=0$ #