2003 初机微 calculus 作业解答

Homework 5 (P.1).

§4.4.

#24. $y = \frac{x^2+x-6}{2x-2} = \frac{1}{2}\left((x+2) + \frac{-4}{x-1}\right)$

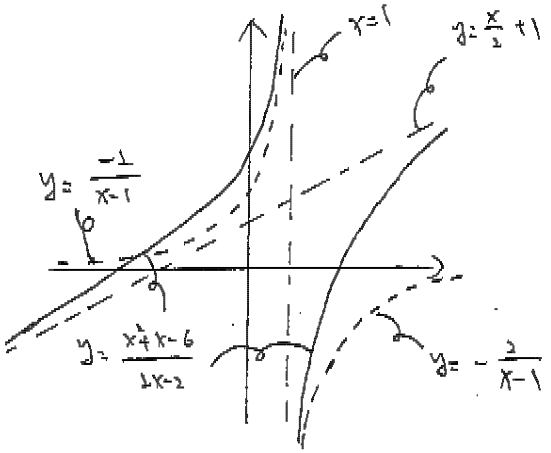
$\Rightarrow y \approx \frac{x+2}{2}$ as $|x|$ large and $y \approx \frac{-3}{x-1} \to \pm\infty$ as x near $1^+, 1^-$

asymptotes:

$x=1$ and $\frac{x}{2}+1$

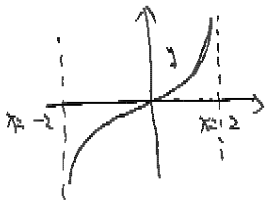
graph:

(过程依照§4.3)



#29. $y = \frac{x}{\sqrt{4-x^2}} = \frac{x}{\sqrt{(2+x)(2-x)}}$

自己做

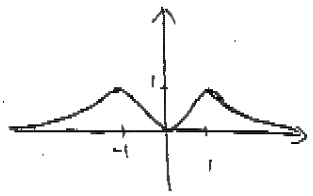


#33. $y = \sin\left(\frac{\pi}{x^2+1}\right) \Rightarrow y \approx 0$ as $|x|$ large.

$y' = -\frac{2x\pi}{(x^2+1)^2}\cos\left(\frac{\pi}{x^2+1}\right) \Rightarrow$ critical pts: $x = 0, 1, -1$.

⋮

自己做.



#41. Assume $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$

$g(x) = b_m x^m + b_{m-1}x^{m-1} + \cdots + b_0$

$\because \lim_{x\to\infty} \frac{f(x)}{g(x)} = 2 \quad \therefore m=n \ \&\ a_n = 2b_m$

Hence $\lim_{x\to-\infty} \frac{f(x)}{g(x)} = 2$

H.W. 5 (p. 2)

§4.4.

#58.

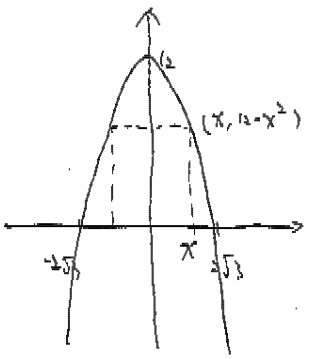
Because the interval about the origin doesn't include enough pixis for such detail.

§4.5.

#3. Suppose perimeter = x in

$\Rightarrow$ Area $= \left(\frac{x}{4}\right)^2 = 16 \Rightarrow x = 16$ in #

#6.



$F(x) = \text{Area} = 2x \cdot (12 - x^2)$

$= 24x - 2x^3$

$F'(x) = 24 - 6x^2 = 6(2-x)(2+x)$

critical point $x = 2, -2$

$F(2) = 32$; $F(-2) = -32$ $\therefore$ Max area = 32 #

#14.

(a) $V(t) = s'(t) = -32t + 96$.

$t = 0$, $V(0) = 96$ (ft/s) #

(b) critical pt. $x = 3$

$s(3) = 256$ (ft)

$\therefore$ max. height is 256 ft #

(c) $s = 0 \Rightarrow t = 7, -1$

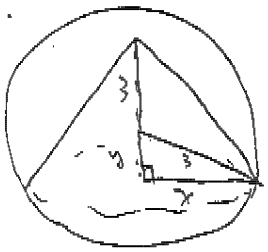
$V(7) = -128$ (ft/s) #

#23. $f'(x) = 2x - \frac{a}{x^2}$

(a) f has local min. at $x = 2 \Rightarrow f'(2) = 4 - \frac{a}{4} = 0$ $\therefore a = 16$ #

(b) $a = -1$ (自己舉例) #

#24.



$y = \sqrt{9 - x^2}$

$\text{Volume} = \frac{\pi x^2}{3}(3 + \sqrt{9 - x^2})$

let $V'(x) = 0$, we have $x = 0, \pm 2\sqrt{3}$.

other critical pt's. $x = \pm 3$.

$V(0) = 0$, $V(2\sqrt{3}) = \frac{32}{3}\pi$, $V(3) = 3\pi$.

$\therefore$ Max volume $= \frac{32}{3}\pi$ #

H.W.5 (P.3)

§4.5.

#27.(a) Suppose width is x in, then depth $= \sqrt{144-x^2}$ in.

Area $= x\cdot(\sqrt{144-x^2})^2 = 144x - x^3$

$A'(x) = 144 - 3x^2 \Rightarrow$ critical pts are $x = \pm 4\sqrt{3}$.

$\therefore$ width $= 4\sqrt{3}$ in, depth $= 4\sqrt{6}$ #

#33. $(\text{distance})^2 = (x-\frac{3}{2})^2 + x$

$= x^2 - 2x + \frac{9}{4} = d(x)$

$d'(x) = 2x - 2 \Rightarrow x = 1$ (critical pt)

$\therefore$ min. distance $= \frac{\sqrt{5}}{2}$ #

#36. (a) f is periodic with period 2π.

(b) $f(x) = \cos 2x + 4\cos x + 3$

$= 2\cos^2 x - 1 + 4\cos x + 3$

$= 2\cos^2 x + 4\cos x + 2$

$= 2(\cos x + 1)^2 \geq 0$

Hence "No" #