2003. 10. 9. 动机清 caculus 作業解答.

Homework 4 (P. 1)

1 §3.7. #9. $f(x) = 2x^2+4x-3$, $x_0 = -0.9$,

$f'(x) = 4x+4$. consider $x=a=-1$

$f'(-1) = 0$, $f(-1) = -5$

then the linearization $L(x) = -5$ #

#10. $f(x) = 1+x$, $x_0 = 8.1$

$f'(x) = 1$, consider $x=a=8$

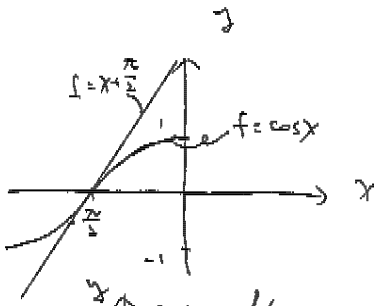
$f(8) = 9$. $f'(x) = 1$

$\Rightarrow L(x) = 9 + 1\cdot(x-8) = x+1$ #

#16. $f(x) = \cos x$, $x = -\frac{\pi}{2}$

$f'(x) = -\sin x$, $f'(-\frac{\pi}{2}) = 1$, $f(-\frac{\pi}{2}) = 0$

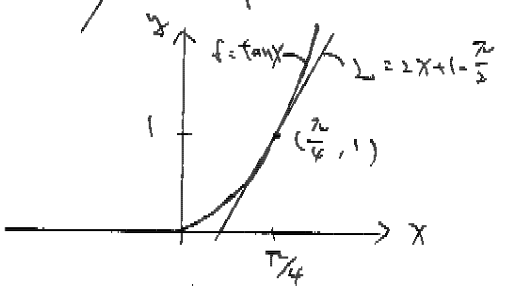
$L(x) = x + \frac{\pi}{2}$



#17. $f(x) = \tan x$, $x = \frac{\pi}{4}$

$f'(x) = \sec^2 x$, $f'(\frac{\pi}{4}) = 2$, $f(\frac{\pi}{4}) = 1$

$L(x) = 1 + 2(x-\frac{\pi}{4}) = 2x - \frac{\pi}{2} + 1$

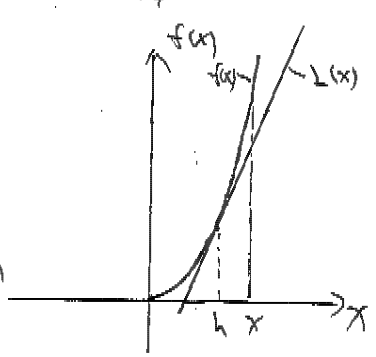


#4.4.

(a) Let $f(x) = \text{volume} = 10\pi x^2$, x: interior diameter.

$f'(x) = 20\pi x$, given $x_0 = h$ fixed

$L(x) = 10\pi h^2 + 20\pi h(x-h)$, is the linearization of f at $x=h$



$\Rightarrow \text{Error} = 20\pi h(x-h) \le \frac{1}{100}(10\pi h^2)$

$\Rightarrow (x-h) \le \frac{h}{200} = \frac{1}{2}\% h$ #

(b) area $= f(x) = 20\pi x$, x = exterior diameter.

$f'(x) = 20\pi$,

$L(x) = 20\pi h + 20\pi(x-h)$, is the linearization of f at $x=h$.

$\Rightarrow \text{Error} = 20\pi(x-h) \le \frac{5}{100} 20\pi h$

$\Rightarrow (x-h) \le \frac{5}{100} h$ #

H.W. 4 (P.2)

§3.8

12. $f(x) = x^{\frac{1}{3}}$ with $x_0 = 1$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^{\frac{1}{3}}}{\frac{1}{3}x_n^{-\frac{2}{3}}}$$

$$= x_n - 3x_n = -2x_n$$

So, $x_1 = -2x_0 = -2$

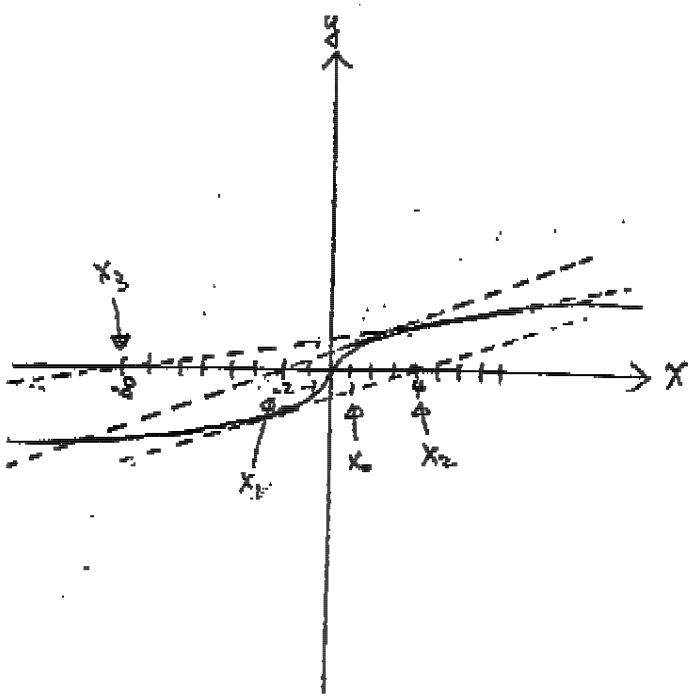
$x_2 = -2x_1 = 4$

$x_3 = -2x_2 = -8$

$x_4 = -2x_3 = 16$

Since $|x_n| = 2x_{n-1} = 2^n$,

$\therefore$ When $n \to \infty \Rightarrow |x_n| \to \infty$



22. $\because \frac{1}{x^2+1} = x$ ←

$\therefore x^3 + x - 1 = 0$

let $f(x) = x^3 + x - 1$

$\therefore f'(x) = 3x^2 + 1$

So, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$= x_n - \frac{x_n^3 + x_n - 1}{3x_n^2 + 1}$$

Hence,

$x_0 = 1$

$x_1 = 0.75$

$x_2 = 0.686046511\,6$

$x_3 = 0.682337582\,6$

$x_4 = 0.682327803\,9$

$x_5 = 0.682327803\,8$

Therefore, 0.68233 is five decimal places.

$d^2 = (x-2)^2 + (x^2 + \frac{1}{4})^2$

$= x^2 - 4x + 4 + x^4 + x^2 + \frac{1}{4}$

$= x^4 + 2x^2 - 4x + \frac{17}{4}$

令 $(d^2)' = 0$, find x ← 極值所在

$4x^3 + 4x - 4 = 0$

$x^3 + x - 1 = 0$

Find x, 即 在 $y = x^2$ 上距離點 $(2,$ 最近的點!

H.W. 4 (P. 3)

7 chap 3

90. (a) let $f(x) = \frac{1}{1+\tan x}$

$$f'(x) = \frac{-\sec^2 x}{(1+\tan x)^2}$$

$\therefore f(0) = 1, \quad f'(0) = -1$

So, the linearization at $x=0$ is

$$y = f(0) + f'(0)\cdot(x-0)$$
$$= \underline{1 - x} \quad \#$$

(b) Let $f(x) = \frac{1}{1+\sin(2x)} \Rightarrow f'(x) = \frac{-2\cos(2x)}{(1+\sin(2x))^2}$

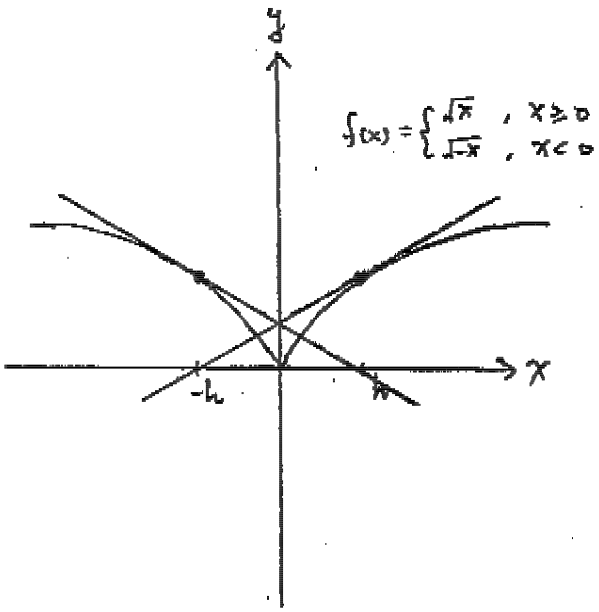
$f(0) = 1, \quad f'(0) = -2$

Hence, the linearization of f at $x=0$ is

$$\underline{y = 1 - 2x} \quad \#$$

§ 3.8

11. $f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$



(i) If $x_0 = h > 0$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = h - \frac{f(h)}{f'(h)}$$
$$= h - \frac{\sqrt{h}}{\frac{1}{2\sqrt{h}}} = h - 2h = -h$$

(ii) If $x_0 = -h, \quad h > 0$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -h - \frac{f(-h)}{f'(-h)}$$
$$= -h - \frac{\sqrt{h}}{\frac{-1}{2\sqrt{h}}} = -h + 2h = h$$

13. $f(1) = -1, \quad f(2) = 8 \Rightarrow f(1)\cdot f(2) < 0$ and

$f(x) = x^3 + 2x - 4$ is conti. $\therefore$ there is a root between 1 and 2. #

$f'(x) = 3x^2 + 2$

$$\therefore x_{n+1} = x_n - \frac{x_n^3 + 2x_n - 4}{3x_n^2 + 1}$$

Let $x_0 = 1$

$x_1 = 1.25$

$x_2 = 1.179judge97$

$\vdots$

$x \simeq 1.17951$ #

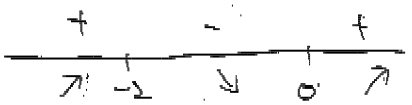
H.W. 4. (P. 4)

§ §4.2

33. $f'(x) = x^{-\frac{1}{3}}(x+2)$

(a) $f'(x) = 0 \Rightarrow x = 0$ or -2 #

(b) $f'(-1) = -1 < 0$, $f'(1) = 3 > 0$, $f'(-8) = 3 > 0$

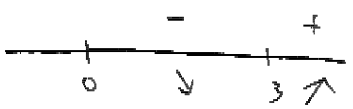
$\therefore$ 

(c) max $x = -1$, min $x = 0$ #

34. $f'(x) = x^{-\frac{1}{2}}(x-3)$, $x \geq 0$.

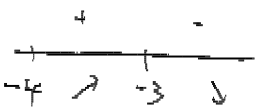
(a) $f'(x) = 0 \Rightarrow x = 0$ or 3.

(b) $f(1) = -2 < 0$, $f(4) = \frac{1}{2} > 0$

$\Rightarrow$ 

(c) max: $x = 0$, min: $x = 3$ #

58. $g(x) = -x^2 - 6x - 9 \Rightarrow g'(x) = -2x - 6$, Let $g'(x) = 0 \Rightarrow x = -3$.

Also, $g'(-4) = 2 > 0$, $g'(0) = -6 < 0$, 

$\therefore g(x)$ has a local maximum at $x = -3$ of value $g(-3) = 0$
local minimum at $x = -4$ " $g(-4) = -1$ #

61. $h(x) = \frac{x^3}{3} - 2x^2 + 4x$, $0 \leq x < \infty$

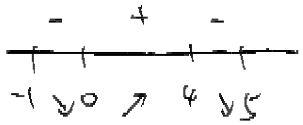
Let $h'(x) = x^2 - 4x + 4 = 0 \Rightarrow x = 2$.

$h'(0) = 4 > 0$, $h'(3) = 1 > 0$

$\therefore h(x)$ has no local maximum.
a local minimum at $x = 0$ of value $h(0) = 0$ #

64. $y = -x^3 + 6x^2$, $-1 \leq x \leq 5$

Let $y' = -3x^2 + 12x = 0 \Rightarrow x = 0, 4$

$y'(-1) = -15 < 0$, $y'(1) = 9 > 0$, $y'(5) = -15 < 0$ $\Rightarrow$ 

$\therefore$ local min: ~~$x = 0$, $y(0) = 0$~~
$x = 0$, $y(0) = 0$
$x = 5$, $y(5) = 25$

local max: $x = 4$, $y(4) = 32$ #
$x = -1$, $y(-1) = 7$

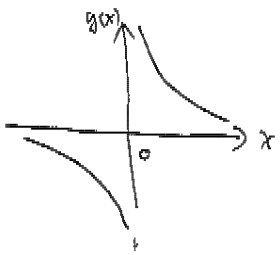
H.W. 4. (P. 5)

§4.2, #81

(a) $g(x) = \frac{1}{x} \Rightarrow g'(x) = -\frac{1}{x^2} \leq 0 \quad \forall\, x \neq 0$.

$\therefore$ $g(x)$ ↓ in its domain. #

(b)



consider the interval $I = (-1, 1)$.

Clearly, g is n.t defined at $x = 0$

$\therefore$ g is decreasing on every interval of $(-1, 0)$, and $(0, 1)$ but not on $(-1, 1)$ #

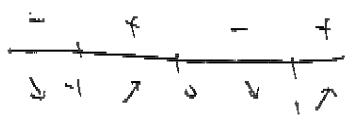
§.4.3 # 17. $y = x^2(x^2 - 2)$

let $y' = 2x(x^2 - 2) + 2x^3 = 2x(2x^2 - 2)$

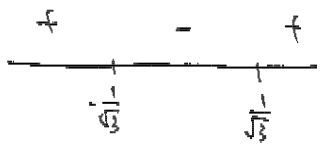
$= 4x(x+1)(x-1) = 0$

$\Rightarrow x = 0, 1, -1$

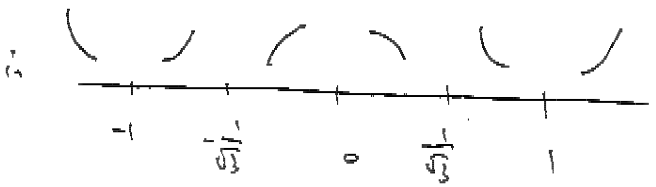
$y'(-2) = -12 < 0$, $y'(\frac{1}{2}) = \frac{3}{2} > 0$, $y'(\frac{1}{2}) = -\frac{3}{2} < 0$, $y'(1) = 12 > 0$



let $y'' = 12x^2 - 4 = 0 \quad x = \pm\frac{1}{\sqrt{3}}$

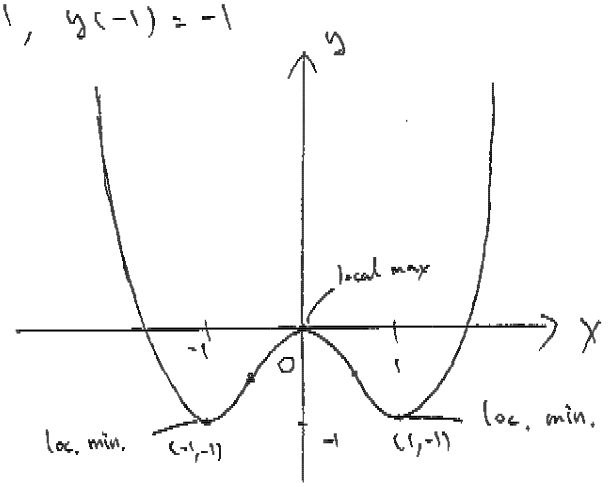


$\therefore$



$y(0) = 0$, $y(1) = -1$, $y(-1) = -1$

$y(\frac{1}{\sqrt{3}}) = y(-\frac{1}{\sqrt{3}}) = -\frac{5}{9}$



H.W. 4 (P. 6)

§ 4.3 # 25.

$y = x + \sin x$, $0 \le x \le 2\pi$.

let $y' = 1 + \cos x = 0$, $x = \pi$ $\qquad y(0) = 0$, $y(\pi) = \pi$, $y(2\pi) = 2\pi$

$y'' = -\sin x = 0$, $x = 0, \pi, 2\pi$.

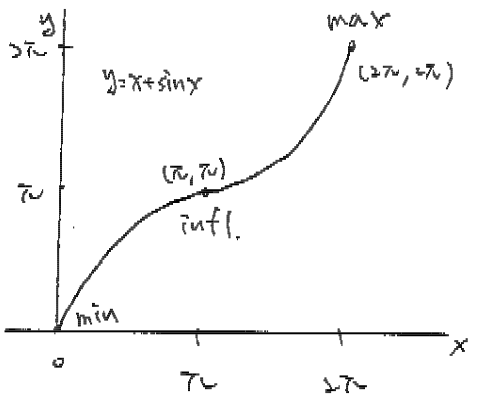
$y'(0) = 2 > 0$, $y'(2\pi) = 2 > 0$, $y''(\frac{\pi}{2}) = -1 < 0$

$y''(\frac{3\pi}{2}) = 1 > 0$

$\Rightarrow$

	0		π		2π
y'		+		+	
y''		−		+	

$\therefore$



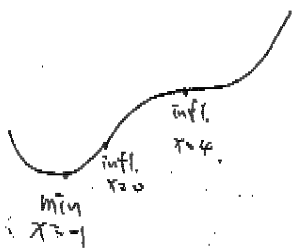
64. $y' = x^{\frac{4}{5}}(x+1)$, $x = 0, -1$

$y'' = x^{-\frac{1}{5}}(\frac{1}{5} - \frac{4}{5x}) = \frac{1}{5}x^{-\frac{1}{5}}(x-4)$, $x = 0, 4$.

$\therefore$

	−1		0		4	
y'	−		+	+		+
y''	+		+	−		+

$\Rightarrow$



76. (a) $y = ax^2 + bx + c$

$y' = 2ax + b$, critical point $x = \frac{-b}{2a}$; $y'' = 2a$, no inflection point

$y(\frac{b}{2a}) = \frac{b^2}{4a} - \frac{b^2}{2a} + c = \frac{-b^2 + 4ac}{4a}$,

$\therefore$ the coordinate of the vertex is $(-\frac{b}{2a}, \frac{-b^2+4ac}{4a})$ #

(b) $a > 0$: concave up $\qquad a < 0$: concave down

		$-\frac{b}{2a}$				$-\frac{b}{2a}$	
y'	−		+	y'	+		−
y''	+		+	y''	−		−

#

H.W. 4 (P.7)

§4.3 #78

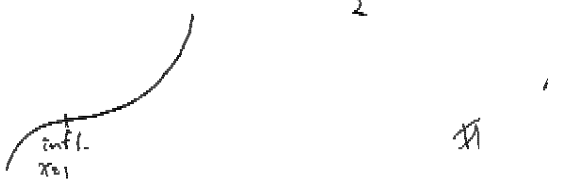
"$\nwarrow$", consider $y' = 3x^4 - 20x^3 + 48x^2 - 48x + 100$.

$y'' = 12(x-1)(x-2)^2$

$\Rightarrow$

		1		2	
y'	+		+		+
y''	−		+		+

$\Rightarrow$ graph of y is



#80. $y = ax^3 + bx^2 + cx + d$, $a \neq 0$

$y' = 3ax^2 + 2bx + c$

$y'' = 6ax + 2b$ $\Rightarrow$ inflection point is $x = \frac{-b}{3a}$