2003. 动机清 caculus 作業解答.

Home work 3 (P. 1)

§3.5.

31. $y = x\tan 3x + 7$

$$\frac{dy}{dx} = \underline{3x\sec^2 3x + \tan 3x} \quad \#$$

32. $y = 2x\sec 2x$

$$\frac{dy}{dx} = 2\sec 2x + 2x\cdot 2\sec 2x\tan 2x$$
$$= \underline{2\sec 2x + 4x\sec 2x\tan 2x} \quad \#$$

33. $y = \frac{1}{21}(3x-2)^7 + \left(4 - \frac{1}{2x^2}\right)^{-1}$

$$\frac{dy}{dx} = \frac{7}{21}(3x-2)^6\cdot 3 + (-1)\left(4-\frac{1}{2x^2}\right)^{-2}\cdot\frac{1}{x^3}$$
$$= (3x-2)^6 - \left(4-\frac{1}{2x^2}\right)^{-2}\cdot x^{-3}$$
$$= \underline{(3x-2)^6 - \left(4x^3 - \frac{x}{2}\right)^{-1}} \quad \#$$

34. $y = (5-2x)^{-3} + \frac{1}{8}\left(\frac{2}{x}+1\right)^4$

$$\frac{dy}{dx} = -3(5-2x)^{-4}\cdot(-2) + \frac{4}{8}\left(\frac{2}{x}+1\right)^3\cdot\left(\frac{-2}{x^2}\right)$$
$$= 6(5-2x)^{-4} - \left(\frac{2}{x}+1\right)^3\cdot\frac{1}{x^2}$$
$$= \underline{\frac{6}{(5-2x)^4} - \frac{\left(\frac{2}{x}+1\right)^3}{x^2}} \quad \#$$

35. $y = x^3(2x-5)^4$

$$\frac{dy}{dx} = 3x^2(2x-5)^4 + 4(2x-5)^3\cdot 2\cdot x^3$$
$$= 3x^2(2x-5)^4 + 8x^3(2x-5)^3$$
$$= \underline{x^2(2x-5)^3(14x-15)} \quad \#$$

36. $y = (1-x)(3x^2-5)^5$

$$\frac{dy}{dx} = (-1)(3x^2-5)^5 + 5(3x^2-5)^4\cdot 6x\cdot(1-x)$$
$$= \underline{(3x^2-5)^4(-33x^2+30x+5)} \quad \#$$

37. $y = (4x+3)^4(x+1)^{-3}$

$$y' = 4(4x+3)^3\cdot 4(x+1)^{-3} + (-3)(x+1)^{-4}(4x+3)^4$$
$$= \underline{(4x+3)^3(x+1)^{-4}(4x+7)} \quad \#$$

38 $y = (2x-5)^{-1}(x^2-5x)^6$

$$y' = (-1)(2x-5)^{-2}(2)(x^2-5x)^6 +$$
$$6(x^2-5x)^5\cdot(2x-5)(2x-5)^{-1}$$
$$= \underline{(x^2-5x)^5(2x-5)^{-2}(22x^2-110x+150)} \quad \#$$

39. $y = \left(\frac{\sin x}{1+\cos x}\right)^2$

$$y' = 2\left(\frac{\sin x}{1+\cos x}\right)\cdot\left(\frac{\cos x(1+\cos x) - (-\sin x)\sin x}{(1+\cos x)^2}\right)$$
$$= \underline{2\frac{\sin x}{(1+\cos x)^2}} \quad \#$$

40. $y = \left(\frac{1+\cos x}{\sin x}\right)^{-1}$

$$y' = (-1)\left(\frac{1+\cos x}{\sin x}\right)^{-2}\cdot\frac{(-\sin x)\sin x - (1+\cos x)\cos x}{\sin^2 x}$$
$$= \underline{2(1+\cos x)^{-2}} \quad \#$$

H.W. 3. (P. 2).

§. 3.5.

61. $y=(1+\tan^4(\frac{t}{12}))^3$

$y'=3(1+\tan^4(\frac{t}{12}))^2\cdot(4\tan^3\frac{t}{12})\cdot(\sec^2\frac{t}{12})\cdot\frac{1}{12}$.

$=\underline{(1+\tan^4(\frac{t}{12}))^2(\tan^3\frac{t}{12}\sec^2\frac{t}{12})}$ #

63. $y=\frac{1}{9}\cot(3x-1)$

$y'=\frac{1}{9}(-\csc^2(3x-1))\cdot 3=-\frac{1}{3}\csc^2(3x-1)$

$y''=-\frac{1}{3}\cdot(2\csc(3x-1))(-\csc(3x-1)\cot(3x-1))\cdot 3$

$=\underline{2\csc^2(3x-1)\cot(3x-1)}$ #

65. $f=u^5+1$, $u=g=\sqrt{x}$, $x=1$

$(f\circ g)'=5u^4\cdot u'=5x^2\cdot\frac{1}{2\sqrt{x}}\Big|_{x=1}=\frac{5}{2}$ #

67. $f=\cot\frac{\pi u}{10}$, $u=g=5\sqrt{x}$, $x=1$

$(f\circ g)'=(-\csc^2\frac{\pi u}{10})\cdot\frac{\pi}{10}u'=-\csc^2\frac{\pi\sqrt{x}}{2}\cdot\frac{\pi}{10}\cdot\frac{5}{2\sqrt{x}}\Big|_{x=1}$

$=(-\csc^2\frac{\pi}{2})\cdot\frac{\pi}{4}=-\frac{\pi}{4}$ #

69. $f=\frac{2u}{u^2+1}$, $u=g=10x^2+x+1$, $x=0$.

$(f\circ g)'=\frac{2u'(u^2+1)-2u'\cdot 2u}{(u^2+1)^2}\Big|_{\substack{u=1\\x=0}}$

$=\frac{2(20x+1)(u^2+1)-2(20x+1)\cdot 2u}{(u^2+1)^2}\Big|_{\substack{u=1\\x=0}}$

$=0$. #

62. $y=\frac{1}{6}(1+\cos^2(7t))^3$

$y'=\frac{1}{6}(3)(1+\cos^2(7t))^2\cdot(2\cos 7t)(-\sin 7t)\cdot 7$

$=\underline{-7(1+\cos^2(7t))^2(\sin(7t)\cos(7t))}$ #

64. $y=9\tan(\frac{x}{3})$

$y'=9\sec^2(\frac{x}{3})\cdot\frac{1}{3}=3\sec^2(\frac{x}{3})$

$y''=3\cdot(2\sec(\frac{x}{3}))(\sec(\frac{x}{3})\tan(\frac{x}{3}))\cdot\frac{1}{3}$

$=\underline{2\sec^2(\frac{x}{3})\tan(\frac{x}{3})}$ #

66. $f=1-\frac{1}{u}$, $u=\frac{1}{1-x}$, $x=-1$

$(f\circ g)'=\frac{1}{u^2}\cdot u'=(1-x)^2\cdot(\frac{1}{(1-x)^2})\Big|_{x=-1}=1$ #

68. $f=u+\frac{1}{\cos^2 u}$, $u=g=\pi x$, $x=\frac{1}{4}$.

$(f\circ g)'=u'+(-2)(\cos^{-3}u)(-\sin u)\cdot u'$

$=\pi+\frac{2\sin u}{\cos^3 u}\cdot\pi\Big|_{u=\frac{\pi}{4}}=5\pi$ #

70. $f=(\frac{u-1}{u+1})^2$, $u=g=\frac{1}{x^2}-1$, $x=-1$

$(f\circ g)'=2(\frac{u-1}{u+1})\cdot(\frac{u'(u+1)-u'(u-1)}{(u+1)^2})\Big|_{\substack{u=0\\x=-1}}$

$=2\cdot\left(\frac{\frac{-2}{x^3}+\frac{-2}{x^3}}{1}\right)\Big|_{x=-1}$

$=8$ #

H.W. 3 (P.3)

§ 3.6

22. $(3xy+1)^2 = 6y$, Find $\frac{dy}{dx}$

$2(3xy+1)\cdot\left(3y + 3x\frac{dy}{dx}\right) = 6\frac{dy}{dx}$

$6y(3xy+1) + 6x(3xy+1)\frac{dy}{dx} = 6\frac{dy}{dx}$

$\therefore \left[6x(3xy+1)-6\right]\frac{dy}{dx} = -6y(3xy+1)$

$\therefore \frac{dy}{dx} = \frac{y(3xy+1)}{1-x(3xy+1)}$ #

26. $x = \sin y$, Find $\frac{dy}{dx}$

$1 = \cos y \cdot \frac{dy}{dx}$

So, $\frac{dy}{dx} = \frac{1}{\cos y} = \sec y$ *

34. $\cos r + \cos\theta = r\theta$, Find $\frac{dr}{d\theta}$

$-\sin r\cdot\frac{dr}{d\theta} - \sin\theta = r + \theta\frac{dr}{d\theta}$

So, $\frac{dr}{d\theta} = \frac{r+\sin\theta}{-\theta-\sin r}$ *

38. $y^2 + 2y = 2x+1$, Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$

(i) $2y\cdot\frac{dy}{dx} + 2\cdot\frac{dy}{dx} = 2 \Rightarrow (y+1)\frac{dy}{dx} = 1$

$\therefore \frac{dy}{dx} = \frac{1}{y+1}$ #

(ii) $\because (y+1)\frac{dy}{dx} = 1$

$\therefore \frac{dy}{dx}\cdot\frac{dy}{dx} + (y+1)\cdot\frac{d^2y}{dx^2} = 0$

So, $\frac{d^2y}{dx^2} = \frac{-\left(\frac{dy}{dx}\right)^2}{y+1}$

$= \frac{-\left(\frac{1}{y+1}\right)^2}{y+1} = -\frac{1}{(y+1)^3}$ *

44. $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$

$2y\cdot\frac{dy}{dx} - 2 - 4\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1}{y-2}$

(a) the slope of the tangent line $m = \frac{dy}{dx}\Big|_{(-2,1)} = \frac{1}{y-2}\Big|_{(-2,1)} = -1$

the tangent line is $y - 1 = -1(x+2)$ #

H.W.3 (P.4)

§3.6.

44. (b) the slope of the normal line at $(-2, 1)$ is

$$\frac{-1}{-1} = 1$$

∴ the normal line is $\frac{y-1}{x+2} = 1$, i.e. $y = x+3$ #

48. $$a = \frac{dv}{dt} = \frac{d}{dt}\left(k\sqrt{s}\right) = \frac{k}{2\sqrt{s}} \cdot \underbrace{\frac{ds}{dt}}_{\| \; v = k\sqrt{s}} = \frac{k}{2\sqrt{s}} \cdot k\sqrt{s} = \frac{k^2}{2}$$

So, the acceleration is constant. #

§3.7

20. $(1+x)^k \approx 1 + kx$

(a) $(1.0002)^{50} \approx 1 + 50 \times 0.0002 = 1.01$

(b) $\sqrt[3]{1.009} = (1.009)^{\frac{1}{3}} \approx 1 + \frac{1}{3} \times 0.009 = 1.003$ #

34. $S = 6x^2$, $x_0 \to x_0 + dx$

$d(6x^2) = 12x\,dx$ ∴ the differential formula of S at x_0 is

$12x_0\,dx$

51. If f has horizontal tangent at $x=a$, then $f'(a) = 0$

So, the linearization of f at $x=a$ is $y - f(a) = f'(a)\cdot(x-a)$

Hence, $y = f(a)$. The linearization is a constant.