2003.9.25. 动机清 caculus 作業解答.

Homework 2 (P.1)

1. chap 2 #37.

$$\lim_{h\to 0}\frac{\sin(\sin h)}{\sin h},$$

$\because \lim_{h\to 0}\sin h = 0$, let $y=\sin h$

$\therefore$ 原式 $= \lim_{y\to 0}\frac{\sin y}{y} = 1$ #

#38.

$$\lim_{y\to 0}\frac{\cos^2 y-1}{\cos y-1} = \lim_{y\to 0}\frac{\cancel{(\cos y-1)}(\cos y+1)}{\cancel{(\cos y-1)}} = \lim_{y\to 0}(\cos y+1) = 2 \ \#$$

#39

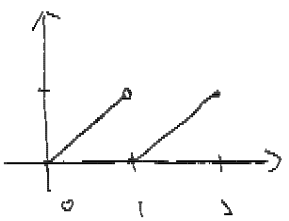
$$\lim_{\theta\to(\frac{\pi}{2})^+}\frac{4\tan^2\theta+\tan\theta+1}{\tan^2\theta+5},$$

$\because \tan\theta\to\infty^+$ as $\theta\to(\frac{\pi}{2})^+$, let $y=\tan\theta$

as #37, then $\lim_{y\to\infty^+}\frac{4y^2+y+1}{y^2+5} = 4$ #

2. False,

consider $f(x)=\begin{cases} x & \text{in } [0,1) \\ x-1 & \text{in } [1,2] \end{cases}$, then $f(x)$ takes any value between $\underset{0}{\underset{\shortparallel}{f(a)}}$ and $\underset{1}{\underset{\shortparallel}{f(b)}}$, but f is not continuous. #



4. §3.1 #35. $f(x)=\frac{1}{\sqrt{x}}$, $c=9$.

$$f'(c) = \lim_{x\to 9}\frac{f(x)-f(9)}{x-9} = \lim_{x\to 9}\frac{\frac{1}{\sqrt{x}}-\frac{1}{3}}{x-9} = \lim_{x\to 9}\frac{3-\sqrt{x}}{3\sqrt{x}}\cdot\frac{1}{x-9}$$

$$= \lim_{x\to 9}\frac{3-\sqrt{x}}{3\sqrt{x}}\cdot\frac{1}{(\sqrt{x}+3)(\sqrt{x}-3)} = \lim_{x\to 9}\frac{-1}{3\sqrt{x}(\sqrt{x}+3)} = -\frac{1}{54} \ \#$$

H.W. 2 (P.2)

4. §3.1 # 40.

(a) $[-2,-1) \cup (-1,0) \cup (0,2) \cup (2,3]$.

(b) $x = -1$

(c) $x = 0, 2$.

49.

(i) $\lim_{t\to 0} g(t)/h(t)$ may exist.

(ii) No.

(iii) Consider $g(x) = x$, and $h(x) = 2x$, then $g(0) = h(0) = 0$

But, $\lim_{x\to 0} \frac{g(x)}{h(x)} = \frac{1}{2} \neq 0$ #

§3.2. # 18.

$$\frac{d(5-x)(4-2x)}{dx} = (-1)(4-2x) + (-2)(5-x) \quad \text{— (a)}$$

$$= -14 + 4x \quad \text{— (b)} \ \#$$

22. $\frac{d(x+\frac{1}{x})(x-\frac{1}{x})}{dx} = \left[1-\frac{1}{x^2}\right]\left[x-\frac{1}{x}\right] + \left[1+\frac{1}{x^2}\right]\left[x+\frac{1}{x}\right]$ — (a)

$$= 2x + \frac{2}{x^3} \quad \text{— (b)} \ \#$$

24. $\frac{d\left(\frac{2x+1}{x^2-1}\right)}{dx} = \frac{2(x^2-1) - 2x(2x+1)}{(x^2-1)^2} = \frac{-2(x^2+x+1)}{(x^2-1)^2}$

26. $\frac{d\left(\frac{x^2-1}{x^2+x-2}\right)}{dx} = \frac{2x(x^2+x-2) - (2x+1)(x^2-1)}{(x^2+x-2)^2} = \frac{x^2-2x+1}{(x^2+x-2)^2} = \frac{(x-1)^2}{(x-1)^2(x+2)^2} = \frac{1}{(x+2)^2}$

28. $\frac{d\left(\frac{(x+1)(x+2)}{(x-1)(x-2)}\right)}{dx} = \frac{[(x+2)+(x+1)](x-1)(x-2) - [(x-1)+(x-2)](x+1)(x+2)}{(x-1)^2(x-2)^2} = \frac{12-6x^2}{(x^2-3x+2)^2}$

32 $\frac{d}{dx}\left(\frac{x}{2\sqrt{x}-7}\right) = \frac{(2\sqrt{x}-7) - x\cdot\frac{1}{\sqrt{x}}}{(2\sqrt{x}-7)^2} = \frac{\sqrt{x}-7}{(2\sqrt{x}-7)^2}$

34 $\frac{d}{dx}\left(\frac{5x+1}{2\sqrt{x}}\right) = \frac{5\cdot 2\sqrt{x} - \frac{1}{\sqrt{x}}\cdot(5x+1)}{4x} = \frac{5\sqrt{x} - \frac{1}{\sqrt{x}}}{4x} = \frac{5x-1}{4\sqrt{x^3}}$

)-1. W. (P. 3)

4. § 3. 2

38 (a) $\frac{dy}{dx} = \frac{(2x+5)x^2 - 2x(x^2+5x-1)}{x^4} = \frac{-5x^2+2x}{x^4} = \frac{-5x+2}{x^3}$

(b) $\frac{d^2y}{dx^2} = \frac{-5(x^3) - 3x^2(-5x+2)}{x^6} = \frac{10x^3 - 6x^2}{x^6} = \frac{10x-6}{x^4}$ #

54

(a) $\frac{d}{dx}(uv)\Big|_{x=1} = u'(1)v(1) + v'(1)u(1) = 0$ #

(b) $\frac{d}{dx}\left(\frac{u}{v}\right)\Big|_{x=1} = \frac{u'(1)v(1) - v'(1)u(1)}{v^2(1)} = 0$ #

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)\Big|_{x=1} = \frac{v'(1)u(1) - u'(1)v(1)}{u^2(1)} = 0$ #

(d) $\frac{d}{dx}(7v-2u)\Big|_{x=1} = 7v'(1) - 2u'(1) = 0$ #

58. (a) $y' = 3x^2 - 3 = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$, then $y = -4$ or 0.

∴ at $(-1, 0)$, the tangent, $y = 0$; the normal, $x = -1$
$(1, -4)$, the tangent, $y = -4$; the normal, $x = 1$ #

68

Ⓐ : $f(x)$

Ⓑ : $f'(x)$ #

69. $\frac{d}{dx}(uvw) = \frac{du}{dx} \cdot vw + \frac{dv}{dx} \cdot uw + \frac{dw}{dx} \cdot uv$.

<pf>: $\frac{d}{dx}(uvw) = \frac{d}{dx}((uv)w) = \left[\frac{d(uv)}{dx}\right] \cdot w + \frac{dw}{dx} \cdot uv$.

$= \left[\frac{du}{dx} \cdot v + \frac{dv}{dx} \cdot u\right] \cdot w + \frac{dw}{dx} uv$

$= \frac{du}{dx} w \cdot v + \frac{dv}{dx} uw + \frac{dw}{dx} uv$. #

H.W. 2 (P.4)

§ 3.4

19. $S = \dfrac{1+\csc t}{1-\csc t}$

$S = \dfrac{1+\frac{1}{\sin t}}{1-\frac{1}{\sin t}} = \dfrac{\sin t+1}{\sin t-1}$

$\therefore \dfrac{dS}{dt} = \dfrac{(\sin t-1)\cdot\cos t-(\sin t+1)\cdot\cos t}{(\sin t-1)^2} = \dfrac{-2\cos t}{(\sin t-1)^2}$ #

20. $S = \dfrac{\sin t}{1-\cos t}$

$\therefore \dfrac{dS}{dt} = \dfrac{(1-\cos t)\cdot\cos t-\sin t\cdot\sin t}{(1-\cos t)^2} = \dfrac{1-(\cos^2 t+\sin^2 t)}{(1-\cos t)^2} = 0$ #

27. $P = \dfrac{\sin\theta+\cos\theta}{\cos\theta}$

$\therefore \dfrac{dP}{d\theta} = \dfrac{\cos\theta\cdot(\cos\theta-\sin\theta)-(\sin\theta+\cos\theta)\cdot(-\sin\theta)}{\cos^2\theta} = \dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$ #

28. $P = \dfrac{\tan\theta}{1+\tan\theta}$

$P = \dfrac{\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}} = \dfrac{\sin\theta}{\cos\theta+\sin\theta}$

$\therefore \dfrac{dP}{d\theta} = \dfrac{(\cos\theta+\sin\theta)\cdot\cos\theta-\sin\theta(-\sin\theta+\cos\theta)}{(\cos\theta+\sin\theta)^2} = \dfrac{\cos^2\theta+\sin^2\theta}{(\cos\theta+\sin\theta)^2} = \dfrac{1}{(\cos\theta+\sin\theta)^2}$ #

53. $\begin{cases} \frac{d}{dx}(\cos x) = -\sin x \\ \frac{d^2}{dx^2}(\cos x) = -\cos x \\ \frac{d^3}{dx^3}(\cos x) = \sin x \\ \frac{d^4}{dx^4}(\cos x) = \cos x \end{cases} \Rightarrow \dfrac{d^{4k}}{dx^{4k}}(\cos x) = \cos x$, for $k \in \mathbb{N}$

Hence, $\dfrac{d^{999}}{dx^{999}}(\cos x) = \dfrac{d^3}{dx^3}\left[\dfrac{d^{996}}{dx^{996}}(\cos x)\right] = \dfrac{d^3}{dx^3}\left[\dfrac{d^{4\cdot 249}}{dx^{4\cdot 249}}(\cos x)\right]$

$= \dfrac{d^3}{dx^3}(\cos x) = \sin x$ #

54. $\begin{cases} \frac{d}{dx}(\sin x) = \cos x \\ \frac{d^2}{dx^2}(\sin x) = -\sin x \\ \frac{d^3}{dx^3}(\sin x) = -\cos x \\ \frac{d^4}{dx^4}(\sin x) = \sin x \end{cases}$ 每微分 4 次做一次循環

即 $\dfrac{d^{4\cdot k}}{dx^{4\cdot k}}(\sin x) = \sin x$ $\because (k = 1, 2, 3, \cdots)$

Hence, $\dfrac{d^{925}}{dx^{925}}(\sin x) = \dfrac{d}{dx}\left[\dfrac{d^{924}}{dx^{924}}(\sin x)\right] = \dfrac{d}{dx}\left[\dfrac{d^{4\cdot 241}}{dx^{4\cdot 241}}(\sin x)\right]$

5. $\frac{d}{dx}\begin{vmatrix} f & g \\ h & k \end{vmatrix} = \frac{d}{dx}(fk - hg) = (f'k + k'f) - (h'g + g'h)$

$= (f'k - h'g) + (k'f - g'h) = \begin{vmatrix} f' & g \\ h' & k \end{vmatrix} + \begin{vmatrix} f & g' \\ h & k' \end{vmatrix}$ #

$= (f'k - g'h) + (f'k - h'g) = \begin{vmatrix} f' & g' \\ h & k \end{vmatrix} + \begin{vmatrix} f & g \\ h' & k' \end{vmatrix}$ #

$$\frac{d}{dx}\begin{vmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{vmatrix} = \frac{d}{dx}\left(f_1 \begin{vmatrix} f_5 & f_6 \\ f_8 & f_9 \end{vmatrix} - f_2 \begin{vmatrix} f_4 & f_6 \\ f_7 & f_9 \end{vmatrix} + f_3 \begin{vmatrix} f_4 & f_5 \\ f_7 & f_8 \end{vmatrix} \right)$$

$$= \left[f_1' \begin{vmatrix} f_5 & f_6 \\ f_8 & f_9 \end{vmatrix} + f_1 \left(\begin{vmatrix} f_5' & f_6' \\ f_8 & f_9 \end{vmatrix} + \begin{vmatrix} f_5 & f_6 \\ f_8' & f_9' \end{vmatrix} \right) \right] - \left[f_2' \begin{vmatrix} f_4 & f_6 \\ f_7 & f_9 \end{vmatrix} + f_2 \left(\begin{vmatrix} f_4' & f_6' \\ f_7 & f_9 \end{vmatrix} + \begin{vmatrix} f_4 & f_6 \\ f_7' & f_9' \end{vmatrix} \right) \right]$$

$$+ \left[f_3' \begin{vmatrix} f_4 & f_5 \\ f_7 & f_8 \end{vmatrix} + f_3 \left(\begin{vmatrix} f_4' & f_5' \\ f_7 & f_8 \end{vmatrix} + \begin{vmatrix} f_4 & f_5 \\ f_7' & f_8' \end{vmatrix} \right) \right]$$

$$= \begin{vmatrix} f_1' & f_2' & f_3' \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ f_4' & f_5' & f_6' \\ f_7 & f_8 & f_9 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7' & f_8' & f_9' \end{vmatrix}$$ #

Similarly $\begin{vmatrix} f_1' & f_2 & f_3 \\ f_4' & f_5 & f_6 \\ f_7' & f_8 & f_9 \end{vmatrix} + \begin{vmatrix} f_1 & f_2' & f_3 \\ f_4 & f_5' & f_6 \\ f_7 & f_8' & f_9 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3' \\ f_4 & f_5 & f_6' \\ f_7 & f_8 & f_9' \end{vmatrix}$ #