2003. 9. 17 动机清 caculus 作業詳解

Homework 1 , (P. 1)

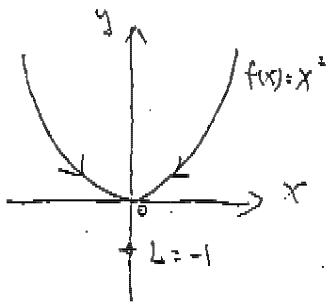
1. Sec. 2.5 # 41

If $f(x)$ gets closer to L as x approches x_0

then the number L is the limits of $f(x)$.

let $L=-1$, & $f(x)=x^2$ & $x_0=0$

then $f(x)=x^2$ gets closer to $L=-1$ as x approches $x_0=0$



But, $\lim_{x\to x_0} f(x) = \lim_{x\to 0} x^2 = 0 \neq L = -1$

Hence, the statement about limits is wrong.

Sec. 2.5 # 42.

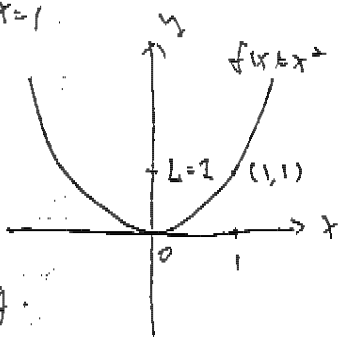
If given any $\varepsilon>0$, there exists a value of x for which $|f(x)-L|<\varepsilon$, then the number L is the limit of $f(x)$ as x approches x_0.

let $L=1$, & $f(x)=x^2$ & $x_0=0$,

then given any $\varepsilon>0$, $\exists$ a value of $x=1$

s.t. $|f(1)-L| = |1^2-1| = 0 < \varepsilon$.

But $\lim_{x\to x_0} f(x) = 0 \neq 1 = L$



Hence, the statement about limit is wrong.

H. W. 1 (P. 2)

1. Chap. 2 #14.

Given $\varepsilon = 0.1$, find $\delta = ?$ $(f(x) = |x|)$

$|f(x) - 0| < \varepsilon \Rightarrow ||x| - 0| < 0.1$

$\Rightarrow |x| < 0.1 \Rightarrow |x - 0| < 0.1$

take $\delta = 0.1$, for $|x-0| < \delta \Rightarrow |f(x)-0| < \varepsilon$. #

Chap. 2 #15.

$f(x) = 5x - 10$, $x_0 = 3$, $\varepsilon = 0.05$, 求 L & $\delta = ?$

$L = \lim_{x \to x_0} f(x) = \lim_{x \to 3} (5x - 10) = 5$

$|f(x) - L| < \varepsilon \Rightarrow |(5x-10) - 5| < 0.05$

$\Rightarrow 5|x - 3| < 0.05 . \Rightarrow |x - x_0| < 0.01$

let $\delta = 0.01$ for $|x - x_0| < \delta$ to imply $|f(x) - L| < \varepsilon$. #

2. a. $\lim_{x \to c^+} f(x) = L$,

Given $\varepsilon > 0$, $\exists\, \delta > 0$, $\Rightarrow$ for $c < x < c + \delta \Rightarrow |f(x) - L| < \varepsilon$. #

b. $\lim_{x \to c} f(x) = \infty$,

Given $M > 0$ $\exists\, \delta > 0 \Rightarrow$ for $|x - c| < \delta$, $\Rightarrow f(x) > M$. #

c. $\lim_{x \to -\infty} f(x) = L$

Given $\varepsilon > 0$, $\exists\, m < 0 \Rightarrow$ for $x < m \Rightarrow |f(x) - L| < \varepsilon$ #

3. Sec. 2.3 #14.

$\lim_{x \to 0} \frac{\sin 5x}{\sin 4x}$

Ans: $\lim_{x \to 0} \frac{\sin 5x}{\sin 4x} = \lim_{x \to 0} \frac{\frac{\sin 5x}{5x} \cdot 5x}{\frac{\sin 4x}{4x} \cdot 4x} = \lim_{x \to 0} \frac{\frac{\sin 5x}{5x} \cdot 5}{\frac{\sin 4x}{4x} \cdot 4}$

$= \frac{5}{4} \cdot \frac{\lim_{x \to 0} \frac{\sin 5x}{5x}}{\lim_{x \to 0} \frac{\sin 4x}{4x}} = \frac{5}{4}$ # (where $\lim_{x \to 0} \frac{\sin 5x}{5x} = \lim_{x \to 0} \frac{\sin 4x}{4x} = 1$

H. W. 1 (P. 3)

3. Sec. 2.3. # 40.

Find $\lim_{x\to\infty} \frac{[x]}{x}$ and $\lim_{x\to-\infty} \frac{[x]}{x}$

Ans: (i) when $x > 0$

$\because \frac{x-1}{x} < \frac{[x]}{x} \le 1$ & $\lim_{x\to\infty} \frac{x-1}{x} = \lim_{x\to\infty} 1 = 1$

by the Sandwich Theorem

$\therefore \lim_{x\to\infty} \frac{[x]}{x} = 1$ #

(ii) when $x < 0$

$\because 1 \le \frac{[x]}{x} < \frac{x-1}{x}$ & $\lim_{x\to-\infty} 1 = \lim_{x\to-\infty} \frac{x-1}{x} = 1$

by the Sandwich Theorem

$\therefore \lim_{x\to-\infty} \frac{[x]}{x} = 1$ #

Sec. 2.3 # 46.

(1) No, because the question doesn't say that $g(x)$, $f(x)$, $h(x)$ are conti. at $x = 2$.
So. $g(2)$, $f(2)$, $h(2)$ can be any value.

(2) Maybe.

(3) No, $\because g(x) \le f(x) \le h(x)$, $x \neq 2$
& $\lim g(x) = \lim h(x) = -5$

Chap 2. #41. $\lim_{x\to0^+} \frac{\sin[x]}{x} = \lim_{x\to0^+} \frac{0}{x} = \lim_{x\to0^+} 0 = 0$

($\because$ When $0 < x - 0 < 1$, $\sin[x] = \sin 0 = 0$)

#42. $\lim_{x\to0^+} \frac{[\sin x]}{x} = \lim_{x\to0^+} \frac{0}{x} = \lim_{x\to0^+} 0 = 0$

($\because$ When $0 < x - 0 < \frac{\pi}{2}$, $[\sin x] = 0$)

#53. $\because 0 \le |\sqrt{x} \sin\frac{1}{x}| \le \sqrt{x}$, for all x, and

$\lim_{x\to0^+} \sqrt{x} = 0$

By sandwich thm

$\therefore \lim_{x\to0^+} \sqrt{x} \sin\frac{1}{x} = 0$ #

H. W. 1 (P. 4)

4. (i) "$x \to c^+$"

Let $P > 0$, Suppose that, for all $c < x < c+P$,

$h(x) \le f(x) \le g(x)$

If $\lim_{x\to c^+} h(x) = L$ & $\lim_{x\to c^+} g(x) = L$,

then $\lim_{x\to c^+} f(x) = L$. #

(ii) "$x \to \infty$"

Given $M > 0$, s.t. for all $x > M$,

$h(x) \le f(x) \le g(x)$

If $\lim_{x\to\infty} h(x) = L = \lim_{x\to\infty} g(x)$, then $\lim_{x\to\infty} f(x) = L$ #

5. Sec. 2.4.

#54. let $g(x) = \begin{cases} 1, & x > -1 \\ 0, & x \le -1 \end{cases}$

$\because \lim_{x\to -1^+} g(x) \neq \lim_{x\to -1^-} g(x)$

$\therefore \lim_{x\to -1} g(x)$ 不存在 $\Rightarrow$ $g(x)$ is not conti. and not removable at $x = -1$.

(Def: $f(x)$ is removable at $x=a \iff \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) \neq f(a)$)

#55. let $f(x) = 1$, $g(x) = x - \frac{1}{2}$ be conti. on $[0,1]$

But $f(x)/g(x) = \frac{1}{x-\frac{1}{2}}$ is not conti. at $x = \frac{1}{2}$

#56. let $f(x) = x$, $g(x) = \frac{1}{x}$, then $h(x) = f(x)\cdot g(x) = 1$ is conti. at $x = 0$

But $g(x) = \frac{1}{x}$ is not conti. at $x = 0$

#57. Yes!

(反證法) If a continuous function is never zero, but it changes sign!

w.l.o.g. (不是一般性 失) 令 $a < b$ & $f(a) < 0$, $f(b) > 0$

then $f(a)\cdot f(b) < 0$

By 勘根 Thm, $\exists\, c \in (a,b)$,

s.t. $f(c) = 0$ $\to\leftarrow$

5. Chap 2. H.W. 1. P.5

#69. $\cos x = 2 - x^3$

$\cos x - 2 + x^3 = 0$

let $f(x) = \cos x - 2 + x^3$

$\because f(100) > 0$, $f(-100) < 0$, $\therefore f(100) \cdot f(-100) < 0$

By 勘根 thm., $\exists\ c \in (-100, 100) \Rightarrow f(c) = 0$

ie. $\cos c - 2 + c^3 = 0 \Rightarrow \cos c = 2 - c^3$.

Hence, $\cos x = 2 - x^3$, has a solution. #

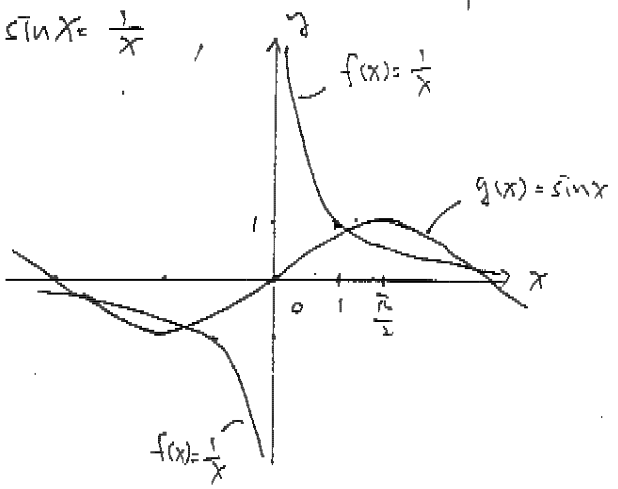
#70. Let $f(x) = x\sin x - 1$

$\because f(\frac{\pi}{2}) = \frac{\pi}{2} - 1 > 0$, $f(\frac{3\pi}{2}) = -\frac{3\pi}{2} - 1 < 0$ $\therefore f(\frac{\pi}{2}) \cdot f(\frac{3\pi}{2}) < 0$

then $\exists\ c \in (\frac{\pi}{2}, \frac{3\pi}{2}) \Rightarrow f(c) = 0 = c\sin c - 1$

Hence, $x\sin x = 1$ has a solu. #

Consider $\sin x = \frac{1}{x}$,



$\therefore x\sin x = 1$ has infinite solutions. #

6. Since f is continuous at $x = c$, and

$2f = f + f$.

By the algebraic properties of continuous functions

Hence $2f$ is continuous at $x = c$ #

3 Sec. 2.3

#34. $\lim_{t\to\infty} t \tan\frac{2}{t} = \lim_{t\to\infty} t \cdot \frac{\sin\frac{2}{t}}{\cos\frac{2}{t}} = \lim_{t\to\infty} \frac{\sin\frac{2}{t}}{\frac{1}{t}} \cdot \cos\frac{2}{t}$

$= 2$ #