Extreme Values of Functions and Graphing

Def: local maximum, minimum
absolute maximum, ~~absolute~~ minimum

Prop: If f is continuous on $[a, b]$ (closed interval)
then f takes an absolute max and absolute min

Possible extreme points

(a) interior points where $f' = 0$

(b) interior points where f' does not exist

(c) end points

Def: (a) and (b) are called critical points

Remark: If not on a closed interval,
the extreme values need not exist

Example: $f(x) = x$ on $(0, 1)$

Methods of finding ~~extr~~ extreme values of continuous function.

Prop: If f has a local max or local min at an interior point c

And If $f'(c)$ exists,

Then $f'(c) = 0$

pf: Use linear approximation

Prop: f is continuous on $[a,b]$ and differentiable on (a,b)

Then (1) $f'(x) > 0$ for all x in (a,b)

$\Rightarrow$ f is increasing on $[a,b]$

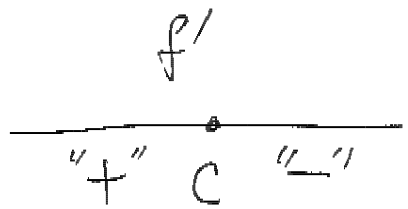
(2) $f'(x) < 0$ for all x in (a,b)

$\Rightarrow$ f is decreasing on $[a,b]$

Remark: "$\Leftarrow$" ~~may not~~ does not hold (can't exclude "$f' = 0$")

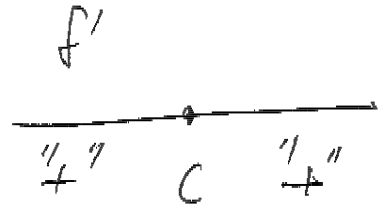
Application

At critical point c



$\Rightarrow$ local max

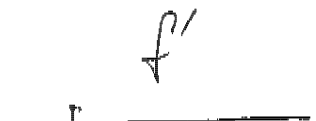
$\Rightarrow$ local min



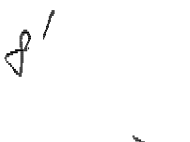
$\Rightarrow$ no local extreme

or "−" "−"

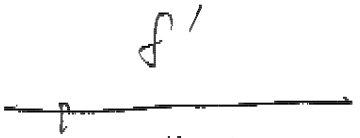
End point



or



$\Rightarrow$ local max



or

$\Rightarrow$ local min

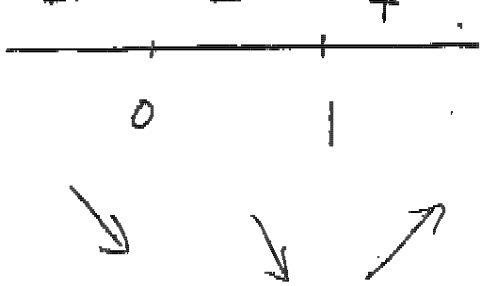
Example $f(x) = x^{\frac{1}{3}}(x-4)$

find local extremes of f

(1) $f'(x) = \frac{4}{3} x^{-\frac{2}{3}} (x-1)$

critical points: $x = 0, 1$

(2) f'



(3) so f decreasing on $(-\infty, 1)$

and increasing on $(1, \infty)$

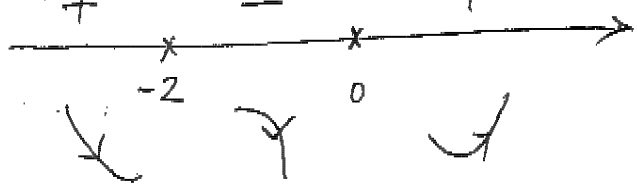
Thus $x = 1$ is a local min and absolute min as well

(4) $f''(x) = \frac{4}{9} x^{-\frac{5}{3}} (x+2)$

$x^{-\frac{5}{3}}$	−	−	+
$x+2$	−	+	+
f''	+	−	+

Concave up on $(-\infty, -2)$, $(0, \infty)$

Concave down on $(-2, 0)$



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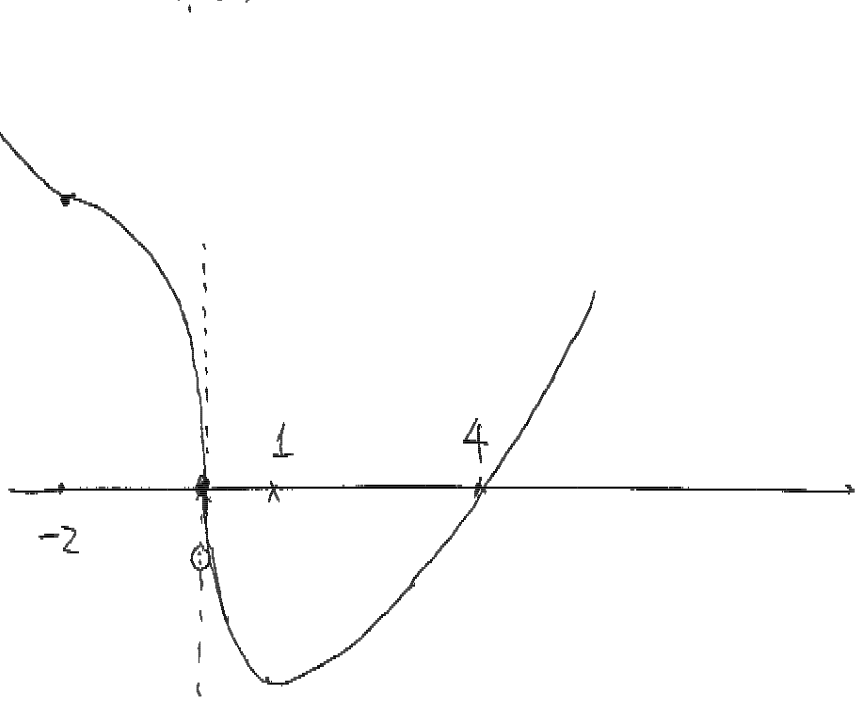
The Graph of f can be concluded from

(1), (2), (3) (4)