Study guide for quiz 10

Quiz problems include both the lecture contents and homework problems.

1. Section 15.5, 15.7:

   General procedure for iterated triple integrals:

Step 1: Draw a cross section: make a cut of "the third (outer most) variable = constant". For example: make a cut of "x = constant" in $dy\,dz\,dx$.

Step 2: Draw lines in the direction of the first (inner most) variable on this cross section to determine the upper and lower limits of integration for the first variable. For example: draw lines in the y direction (i.e, parallel to y-axis) in $dy\,dz\,dx$.

Step 3: Move these lines in Step 2 in the direction of the second (middle) variable on this cross section to determine the upper and lower limits of integration for the second variable. For example: move lines in Step 2 in the z direction in $dy\,dz\,dx$.

Step 4: Move these cross sections in the third (outer most) variable direction to determine the upper and lower limits of integration for the third variable. For example: move the cross sections from Step 1 in the x direction in $dy\,dz\,dx$.

2. Section 15.7: Study the meanings of the variables ρ, θ and ϕ, respectively. Study and memorize the change of variables formula between (x, y, z) and (ρ, θ, ϕ) (both ways).

3. Section 15.7:

   Practice on drawing cross section $\{\rho = \text{constant}\}$, $\{\theta = \text{constant}\}$ and $\{\phi = \text{constant}\}$ in Spherical coordinates. Which one is needed for $d\rho d\theta d\phi$? which one is needed for $d\phi d\rho d\theta$? etc.

   On a cross section, the triple integral reduces to double integral for the first two integration variables. For example, on a $\{\theta = \text{constant}\}$ cross section, it reduces to double integral $d\rho d\phi$. One can then determine the upper and lower limits of integration as outlined above.

4. Section 15.7, 15.8:

   Memorize the Jacobian for Cylindrical Coordinates and Spherical Coordinates, respectively.

5. Section 15.8:

   Study the meaning of the Jacobian and memorize the formula both in double and triple integration.

6. Section 15.8:

   General procedure for Substitution in double integrals:

Step 1: Given the new variables $u = u(x, y)$, $v = v(x, y)$. Solve for the inverse formula $x = x(u, v)$, $y = y(u, v)$ and use it to compute the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

Step 2: Follow standard procedure, from the limits of integration in x and in y, plot R, the region of integration in (x, y) plane.

Step 3: Find the equations (in x and y) for the boundary curves of R in (x, y) plane. Use the inverse formula in Step 1 to rewrite these equations in terms of (u, v) and use them to plot the region of integration in (u, v) plane.

Step 4: Follow standard procedure to determine the limits of integration in $du\,dv$ or in $dv\,du$.

7. Section 15.8:

   Suppose that a change of variables between (x, y, z) and (u, v, w) is given, study how to change the lower and upper limits of integration for (x, y, z) into lower and upper limits of integration for (u, v, w). See the examples in section 15.8.