Study guide for quiz 07

Quiz problems include both the lecture contents and homework problems.

1. Section 14.3-14.5:

   Review definition(s) and necessary and/or sufficient conditions of "$f(x, y)$ is differentiable at (x_0, y_0)". See Remark_on_definition_of_differentiability_v04.pdf, Exercise 2 (the easy ones in Exercise 2, at least).

   (v04 is the newest version with Exercise 2 added)

2. Section 14.6:

   Review the definition and properties of the gradient vector and its application in finding the tangent plane and normal line at a point (x_0, y_0, z_0) on the level surface $F(x, y, z) = c$ of a differentiable function F. Review how to find the tangent line and normal plane at a point (x_0, y_0, z_0) on a curve in space defined as the intersection of two surfaces $F(x, y, z) = C_1$ and $G(x, y, z) = C_2$.

3. Section 14.6:

   Study and memorize the error estimate of $E(x, y) = f(x, y) - L(x, y)$ for a function $z = f(x, y)$ and its linearization $z = L(x, y)$ at a point (x_0, y_0). Study its generalization to functions of more than two variables (i.e. $g(x, y, z)$, $h(x, y, z, w)$, $\cdots$, etc.).

4. Section 14.6:

   SKIP the 'differential' part on page 858, which reads:

   $$df = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy.$$

   Instead, you need to memorize the formula below and understand its meaning:

   $$\Delta f \approx f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y.$$

5. Section 14.7:

   Study the 'First Derivative Test', 'Second Derivative Test', how the sign of the determinant $f_{xx}f_{yy} - f_{xy}^2$ is related to whether $f_{xx}\Delta x^2 + 2f_{xy}\Delta x\Delta y + f_{yy}\Delta y^2$ can be rewritten as 'sum of squares' or 'difference of squares', which in term determines whether a critical point is local min, local max, or neither. Review the procedure of finding potential local minima and/or local maxima of a differentiable function $z = f(x, y)$.

6. Section 14.7:

   Review the gradient analysis that helps to classify the critical points when the second derivative test is inconclusive, and how it can be applied to find absolute max and absolute min on a bounded region.